Universities are designing new courses and licensure programs to support the enactment of reform recommendations by prospective teachers. Professional development schools provide a context in which prospective teachers collaborate with teachers and university professors to connect theory and practice. Teachers’ experiences in mathematics often do not reflect current reform recommendations and enacting new pedagogy can be problematic for them. This study sought to explore how cognitive dissonance may help prospective teachers make their implicit conceptions about teaching and learning explicit and support their reconstruction of these notions. Findings indicate that elementary teachers’ conceptions about mathematics change after experiencing and reflecting on cognitive dissonance.

Despite calls for reform in school mathematics by The National Council of Teachers of Mathematics (NCTM), classroom practices in United States during the last century have shown little change (Stigler & Hieber, 1999). The University of Colorado at Denver and Health Sciences Centre created a licensure program using professional development schools (PDS) to increase the number of highly qualified teachers with the leadership skills needed to support educational reform. A PDS is a collaborative community of learning which includes prospective teachers (approximately 12), teachers, administration, and a site professor who supports educational reform through school-wide professional development.

Prospective teachers work within the PDS to gain practical experiences while simultaneously developing their content and pedagogical knowledge in university courses. These beginning teachers gradually influence classroom practices at their PDS as they examine and discuss the influence of a teacher’s practices with students’ learning at weekly site seminars. Annotative stories told by faculty indicate that many prospective teachers eventually assume leadership positions in their school, school district, and the State of Colorado.

Current research on the development of elementary teachers concentrates on the growth of content (e.g., Lo, 2004; Southwell & Penglase, 2005), pedagogical (e.g., Leu & Wo, 2005), or an integration of content and pedagogical knowledge (e.g., Ball & Bass, 2000; Beswick, 2005). This research suggests that prospective teachers have idiosyncratic knowledge of mathematics content and implement reform recommendation that resembles traditional practices. Beswick investigated prospective elementary teachers’ conceptions of relational and instrumental understanding. She suggested that prospective teachers need powerful evidence to
create viable learning experiences that are different from the ones that they themselves experienced as a student. Beswick theorized that these new learning experiences develop a different kind understanding. These suggestions reflect Ball and Bass’s assertion that content and pedagogy must be interwoven as perspective teachers learn to teach. This study extends the work of these researchers by examining experiences that support prospective elementary teachers’ growth over a three year period of time and enable them to assume leadership roles during their early career.

THEORETICAL FRAMEWORK

From the perspective of symbolic interactionism, individuals learn as they interact with other people and the environment (Blumer, 1969). Interactions between individuals are exchanges of words, tone of voice, facial expressions, and gestures that allow individuals to co-create knowledge by sharing ideas, questioning assumptions, and clarifying interpretations. Taylor (2001) suggested that the mediation of verbal and non-verbal interactions may transform the concealed implicit learning into articulated explicit ideas. He inferred that the interaction of implicit and explicit experiences is critical for learning to be transformative. From this stance, Olson, Chiado, Sala, & Kirtley (2005) theorized that (a) the transformation from implicit to explicit may promote teachers’ self-efficacy, which emerges through critical reflection of deeply held beliefs and emotions and (b) an increase in teachers’ self-efficacy enables teachers to assume leadership roles.

Olson, Chiado, Sala, & Kirtley (2005) created a model to describe the relationships between formal and informal experiences with the process of transforming implicit conceptions to explicit conceptions. Implicit conceptions are difficult for individuals to express and encompass many emotions that arise from prior learning experiences. Transforming conceptions is a complex reflective process. Formal and informal learning experiences are interwoven into conceptions about teaching and learning. Olson et al. theorize that implicit conceptions impact how an individual interprets new experiences and assimilates new beliefs. They suggest cognitive dissonance may allow individuals to make their implicit conceptions explicit and examine their implicit conceptions from a new light. When Kirtley reflected on whether her practice reflected a newly articulated belief, she experienced a moment of revelation and changed in her practice (Olson & Kirtley, 2005).

The model suggests that cognitive dissonance is one way to prompt individuals to critically examine their implicit beliefs in light of new experiences and support transformational change. Research using this model to interpret social interactions indicated that teachers began to understand and articulate their implicit learning through discussions that were punctuated by questions that prompted critical reflection. Olson et al. suggest that this critical reflection increases self-efficacy by helping teachers connect theory with practice and gain confidence. From the perspective of symbolic interactionism, when self-efficacy increases, teachers’
interactions with colleagues change as they voice their ideas in new ways and may lead to new informal or formal leadership roles.

**RESEARCH DESIGN AND APPROACH**

A multilevel research design merges the structure of the multi-tiered teaching experiment and case study to describe how collaboration develops the knowledge of the participants at different levels of learning (English & Watters, 2005). This multilevel research study used two phases. The first phase focused on the learning of prospective elementary teachers while they participated in mathematics licensure courses and field work in a PDS. The second phase began after the prospective teachers gained licensure to teach elementary school and decided to complete their Master’s Degree in mathematics education.

**Phase 1**

At the first level of phase 1, the prospective elementary teachers solved non-routine problems and planned instructional lessons in collaborative groups. Groups consisted of four prospective teachers with similar beliefs about the nature of teaching and learning mathematics and different levels of proficiency in mathematics. Data sources at the first level included: written reflections, written solution strategies, and Olson’s field notes made while teaching mathematics education courses. These data were analysed for changes in the teachers’ justification, creation of conjectures, and spontaneous articulation of mathematical ideas.

At the second level, the prospective teachers worked in PDS with a clinical teacher to plan and teach mathematics lessons to elementary students. The prospective teachers videotaped the lessons and collected student work samples for analysis. Using developmental frameworks (e.g., cognitively guided instruction), the prospective teachers analysed these data for evidence of students’ conceptual understanding and learning. Then, they reflected on how their actions influenced elementary students’ opportunities to learn. These student products and class discussions were analysed by Olson for situations that led to cognitive dissonance. Changes in what teachers noticed or analysed were interpreted as evidence of learning.

**Phase 2**

The second phase of this study began after the prospective teachers gained licensure to teach elementary school. Colasanti and Trujillo decided to complete their Master’s Degree in mathematics education and were selected for case-study analysis. They entered the licensure program with different levels of confidence and expertise in mathematics and began to collaborate during a course on rational numbers when they were hired to teach in the same elementary school. Colasanti and Trujillo continued to collaborate throughout their first two years of teaching.

At level one, Colasanti and Trujillo’s elementary students solved problems selected from a reform curriculum. The two teachers monitored their students’ learning and met with two school district math coaches to discuss the development of students’ understanding. In addition, they explored mathematical ideas during these
discussion and how their actions impacted student learning. Colasanti and Trujillo wrote reflections about these coaching sessions, collected student work samples, and wrote field notes while teaching. These data were analysed to describe the process by which they connected theory with practice and reported in their Master’s Projects.

At level two, Colasanti and Trujillo participated in graduate courses to complete their Master’s Degree and solved non-routine problems designed to help them deepen their understanding of mathematical ideas. These courses focused on rational numbers, ethnomathematics, and mathematical modelling in which content and pedagogy were intertwined. Elementary and Secondary teachers worked together while developing content knowledge and worked in grade level groups while creating lessons. Data sources included: written reflections, written solution strategies, and Olson’s field notes. Data were analysed for changes in teachers’ level of sophistication in their mathematical arguments, rationale for their lessons, and analysis of students’ work samples.

At level three, Olson, Colasanti, and Trujillo analysed the learning that occurred at level one and two in both phases for situations that led to cognitive dissonance. We examined the collected data for patterns that connected (a) graduate course work with articulated beliefs about teaching and learning, (b) experiences that led to cognitive dissonance, (c) discussions about the dissonance, with (d) the encouragement to assume new leadership roles within the school.

RESULTS AND DISCUSSION

Colasanti and Trujillo both remembered experiencing elementary mathematics as a series of facts and procedures to memorize (reflections, September 07, 2003). However, Colasanti’s father enjoyed mathematics and she remembered him “emphasizing that understanding what I was doing was going to make it a lot easier as math got more complicated.” Colasanti internalized an image of herself as a “doer” of math when she understood WHY and this led to understanding. In contrast, Trujillo experienced early frustration with math. She believed that math was comprised of abstract ideas that she would never understand and recalled being ridiculed in high school for using an incorrect strategy. Through these experiences, Trujillo came to “hate” math and was “deathly afraid of teaching math” in the elementary school (reflections, August 24, 2003).

From these experiences and emotions, a notion of teaching and learning mathematics was constructed that was comprised of both implicit and explicit conceptions. Both Colasanti and Trujillo articulated similar teaching goals, “I want to teach in a way so that kids will understand math” and envisioned teaching practice as, “I will show them how to solve problems and explain each step so that they will understand what to do” (reflections, August 24, 2003). Illustrative examples will be presented to describe how cognitive dissonance helped Colasanti and Trujillo make their implicit conceptions explicit by analysing new experiences learning and teaching mathematics.
Phase 1

The licensure program was designed to provide prospective teachers with experiences that explored new ideas while working with elementary students in PDS. Both Colasanti and Trujillo struggled in the introductory mathematics content course for elementary teachers, but for very different reasons. Colasanti reflected, “I never had a problem with math and loved timed tests. I didn’t know why we were learning to use manipulatives. Math was based on symbols. But, suddenly it clicked when I taught geometry. I never got geometry because I am not a visual person” (interview, November 28, 2005). Colasanti experienced cognitive dissonance as she questioned the usefulness of manipulatives in a mathematics class. Then, she discovered that her own ability to conceptualize geometric ideas may have been limited without the use of objects to construct mental images. Colasanti taught a unit on geometric shapes and then “all of a sudden I knew what a quadrilateral was. It wasn’t just a rectangle or something like that. It was a whole group of shapes that included squares and funny looking shapes with four sides, just like we talked about in class.” The experience exploring geometric ideas in a content course and then teaching geometry to third grade students led Colasanti to make her implicit conception that mathematics was symbolic manipulations explicit. She then was able to reconsider this conception and articulated the importance of using manipulatives in classrooms to help students explore characteristics of shapes and construct visual images that can be manipulated in the mind (reflection, March 2, 2004). Mathematics was no longer symbols that were used to get answers.

In contrast, Trujillo avoided mathematics because she “never had a grasp of it” (interview, November 28, 2005). She struggled in the introductory mathematics course for elementary teachers because she “did not have the procedural knowledge to solve math problems.” Trujillo experienced cognitive dissonance when she entered the introductory course and confronted her belief that she would fail because of her limited understanding of mathematics. She discovered that knowing the procedures did not help her colleagues and found that she in fact “could solve problems that other couldn’t solve.” This led her to reconceptualise her self image and began to envision herself as a “math person.” With her new confidence, Trujillo decided to complete her Master’s Degree in Mathematics Education and described herself as an “elementary teachers who enjoyed math and loved to teach it” (interview, April 14, 2005).

Phase 2

Secondary and elementary teachers finishing their Master’s Degree in mathematics education complete a course on the structure of rational numbers. They collaboratively investigate multiple representations of rational numbers to solve non-routine problems (Lamon, 1999), to deepen their understanding of mathematics content, and to explore pedagogy that develops conceptual understanding. During the spring 2004, Colasanti and Trujillo participated in this course on rational number. Olson assigned each teacher to a specific group that mixed the strengths of group members. Initially, Colasanti and Trujillo were in different groups but began
collaborating after Trujillo accepted a mathematics position in the elementary school in which Colasanti had accepted a fifth grade teaching position.

During the second week of the rational number course (field notes, June 15, 2004), Olson asked the teachers to draw a picture to represent \( 8 \div 2 \). Two thirds of the teachers drew a picture of eight objects circles around the two groups of four objects. Colasanti explained, “I drew eight objects and put them into two groups.” Trujillo responded, “That is really neat! I drew eight objects and put two in each group. The answers are the same, four, but what we did was really different.” The teachers in the class began to recognize that unless students are asked to show and explain their thinking, “we may make incorrect assumptions about their mathematical thinking.” This discussion prompted all the teachers in the class to reconsider their implicit conceptions that if students arrived at the same answer then their visualization were also the same. While all of the teachers recognized that different solution strategies often lead to the same correct answer, they had never considered that the way students visualize problems may also differ. Articulating this implicit conception led the teachers to consider how students’ images of mathematical ideas may differ and influence the meaning attached to symbols.

Olson further challenged the teachers by asking them to represent \( 4 \div 1/3 \) using both interpretations and \( 3/8 \div 3 \). The group with Colasanti and Trujillo quickly drew four circles, partitioned each one into thirds, and then counted the number of thirds. To create the measurement model, almost all of the teachers in the class needed a problem context. One elementary teacher conceptualized division using the measurement model and posed the following problem. “Suppose you have four cups (drew four circles on the board) and this is only one third of a jug (drew an oval around the cups). How many cups would be in the jug?” Colasanti immediately responded, “There would be 12. So division is really just multiplying… oh, I see why you invert and multiply.”

Colasanti and the other teachers experienced cognitive dissonance as they struggled to model division of fractions by constructing a unit from a part. They recognized that teaching students algorithms may help them get the right answer. But, unless students understood how the algorithm reflected problem solving situation, they would not be able to apply the algorithm in a problem context. The teachers’ experience of conceptualizing \( 4 \div 1/3 \) using only the partitive model encouraged them to reconsider their own conceptual understanding of division. The process of using cognitive dissonance to reflect on previous experiences and to articulate implicit assumptions mathematical understanding enabled Colasanti and Trujillo to reconsider their notions about teaching and learning mathematics.

Colasanti and Trujillo and continued to collaborate in their elementary school through informal meetings. The focus of these meetings was to discuss “how we were going to implement [teach] specific concepts in our classrooms” (reflection, December 30, 2005). Trujillo noted that “when the students did not have a strong enough foundation to start where the book was, we examined the state standards to extract what concepts needed to be understood. From there we pulled from our own
resources and discussed our plan with our math coaches.” The two district math coaches encouraged Colasanti and Trujillo to consider “why we decided to teach things a certain way” (reflections, January 2, 2006). This built Colasanti and Trujillo self-confidence to use their own knowledge, experiences, and resources to find answers to difficult questions. At times, Colasanti and Trujillo reported that their principal sat in on these meetings. During their second year teaching (2005-2006), the principal began to refer to them questions about mathematics and the staff viewed them as mathematics resources (interview, November 28, 2005).

In summary, cognitive dissonance led Colasanti and Trujillo to transformative change when they reflected on their emotional responses, prior experiences, and new experiences. We suggest that this process of reflection can help teachers articulate their implicit conceptions and that this articulation is critical to support change that is transformative. We also suggest that as teachers experience transformative change their self-efficacy also changes, thus, positioning teachers to assume new leadership roles in their school.

IMPLICATIONS

In conclusion, this study suggests that creating opportunities for teachers to examine their implicit conceptions about teaching and learning mathematics can be accomplished through cognitive dissonance. We describe this process as making the implicit explicit and theorize that not only does this process support reflection but it also can lead to increased self-confidence. Beswick (2005) suggested that new learning experiences can develop a different kind understanding with robust evidence. We found that cognitive dissonance may create emotional and experiential evidence that can be articulated and analysed to make implicit conceptions explicit. This process may be a mechanism that stimulates a belief-system change. Describing this process in which cognitive dissonance is used to uncover conceptions about teaching and learning mathematics may help teacher educators monitor the process of reflection and help them phase questions that promote growth.

Additional research is needed to examine whether this process is effective with secondary and elementary teachers who participate in grade level professional development outside university courses. Longitudinal research is also needed to investigate experiences that support early-career teachers to assume leadership roles in their schools.

References


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