

TEXTUAL DIFFERENCES AS CONDITIONS FOR LEARNING PROCESSES

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The aim of the study (Johnsen Høines 2002) reported here was to shed light on the process through which student teachers express and interpret their understandings about mathematical notions and thereby gain insight into them. This paper focuses on how students cooperate and move between different ways of understandings. It emphases in particular on how differences in the ways of explaining a mathematical content make different conditions for the dialogue and learning processes that develop.

INTRODUCTION

The study to which this paper relates (Johnsen Høines 2002), had the purpose of shedding light on the processes employed by student teachers to communicate mathematical content; on how they do gain insight into mathematical notions by exploring different ways of expressing, interpreting and investigating them. I wanted to learn about those processes whereby students engage in learning in ways that can be described as taking ownership of it (Mellin-Olsen 1989, Skemp 1971, Goodchild 2001). Such situations can be characterised by the behaviours exhibited when students are in charge, making decisions about what to do and how to do it. They develop authorship (Burton 1999). I had, as a teacher and a teacher educator, experienced such processes within situations that were characterised by energy, determination, communication and independence. I wanted to study how the students, within the frame of such situations, move between knowledge and knowing, and how they use *differences* within the ways of expressing content as tool for their goal related activities.

THEORETICAL BACKGROUND

The dialogical perspective on learning referred to in this paper is related to the work of M.M.Bakhtin (1981,1998). Understanding is seen as constituted by ways of understanding. To develop understanding is, in the study on which this paper is based (Johnsen Høines 2002) accordingly seen as moving between ways of understanding, conducting the ways of understanding in opposition to one another, seeing them in the light of the other(s). Bakhtin (1981:12) states that “*Language throws light on each other: one language can, after all, see itself only in the light of another language.*” The differences get vital. Without the differences the interactions would not have any function. The understanding would not develop. “*Different voices are not enough to create meaning; the tension and struggle between them create understanding*” (Dysthe

1999:76) The dialogical approach used in the study is a text-theoretical approach where I see texts as constituted by texts interacting with each other.

METHODOLOGY

The empirical data was gathered from a situation where student teachers were working on a task within the theory of functions:

Five 14-year olds have started a bicycle-club. They repair bicycles, they arrange outings and they discuss traffic problems. The organisers of the club wish to recruit more members in order to make the club grow. They set the goal that each member should recruit three more members every quarter of the year until they reach a set limit of 1000 members. A waiting list is established from which to pick new members should someone drop out. Express the aspired number of members as a function of time and by using the premises set for how to reach the aspired goal.

A particular episode developed in interaction between two of the students, Mette and Kari. It was chosen as a research situation because of four criteria: the students' intentionality, the different ways of understanding that were communicated, the interactions between the students and the way they persisted because they wanted to gain insight into the processes.

I subscribed the communication on the basis of observation. The students were subsequently interviewed. I invited them to comment on the subscript and to tell about what they thought happened in the episode. The interviews and comments were transcribed. These transcripts served as the empirical data for the study.

To get in touch with the complexity underneath, I wanted to make a detailed analysis of a short situation. A text-theoretical perspective underpinned the analyses in that I studied the learning situation as a text. I understand the concept of *text* in a broad way by regarding text as what is interpreted as well as the interpretation. When I refer a written text, text is not only seen as what is written. This is taken together with a reader's interpretation. A text can also be oral or it might be a picture. A (teaching/learning-) situation can also be regarded as text. To summarise, text is conceived as a combination of what is being interpreted together with the interpretation. Texts are developed in interaction between texts through a process of confronting each other (Bakhtin 1981).

The task referred to above is in itself referred to as a text. It serves as an illustration for the individual and social approach. It is individual in the sense that the students' interpretations will differ. Each student has her own interpretation. The social dimension implies that their interpretations are related to the interpretations of the other students' interpretations and of earlier interpretations done.

When analyzing the differences, I found that three aspects achieve importance: *First*, I regard the texts as related to the individual; Kari's text and Mette's text differ. The

students make different texts relevant as a basis for their interpretations, for further developing.

Second, this illustrates the social perspective. They discuss. They have (partly) shared basis for their references. This might be conscious and shared knowledge. They talk, for instance, about “this is how we understand it”. In the context of Bakhtin’s approach, we have social references also without being conscious of them. Our personal interpretations imply social dimensions. Thinking is seen as dialogic; in itself it is social. I look into how the differences between the students’ understandings can be understood in relation to a social dimension.

Third, the students relate to texts that predict a kind of objectivity. The task is formulated in a certain way. It has a specific genre. When reading, the students make other tasks and other kinds of mathematical texts relevant to their interpretations. The task is ordered in a specific way. The processes of understanding can be seen as attempts to make a structure of what is structured; make an order of the order one observes is there. This can also be seen as describing the learners’ struggle to know, by moving between knowing and knowledge. (Burton 1999) According to the concept of genre I relate to in this study, the order carries in itself a content. Ordering a content in different ways, make different meanings. (Bakhtin 1981, Johnsen Høines 2002)

I see the discussion Hewitt (2003) makes about how the visualisation of a mathematical content constitutes challenges for the teaching- learning processes relevant to these aspects. “Learning to use formal notation involves not only developing meaning for symbols but also developing meaning for the positioning of symbols in relation to other symbols. The symbols within an algebraic equation and their relative positions reproduces a visual impact which affects the way those equations are manipulated when re-arranging the equation.” (p.3-68) I learnt from my study, that Kari and Mette saw the content and the way it was expressed as interwoven. They struggled to understand the mathematics by investigating what was expressed by other persons and by themselves. As mentioned above, I regard their way of expressing as a way of ordering the content. There is a content embedded within the way of ordering, decided within the genre. In this paper, when learning is seen as developing text, it is seen relational, related to an individual, social and the genre-related perspective. (Johnsen Høines 2002)

It becomes a methodological challenge to uncover interpretations that are legitimated in the context of the dialogic process, to visualise them in the light of each other and to evaluate them as plausible interpretations from the perspectives of the individual, social and genre related aspects mentioned above. I experience it as challenging and vital to develop the analyses by moving between apparently conflicting aspects: regarding the text as both nearly not personal and personal. This implies moving between my interpretations of what is said and what is written a) as little as possible

in the context of what I know about the person and her intentions and b) in the context of the students, the relationships between them and the personal intentionality embedded in their actions. I find it, within the limitation of this paper, impossible to bring forward the complex analyses that in the study are underpinning my interpretations. When the data is referred to in the following, it is mostly presented as my interpretations and it has to a certain degree an illustrative function.

STRUGGLING TO UNDERSTAND WHY THE ANSWERS DIFFER

The situation turned into a research situation when Kari and Mette communicated: *"We don't understand this. It does not fit. We have different answers and we cannot understand why!"* The students were intensively searching for insight into the problem. They did not ask: *"What is the correct answer?"* They communicated their questioning with the teacher and a few other students just being there. They commented on their failure to understand why the answers differed. They were deeply engaged, looking at each other's written work. They talked slowly in an investigative way, were listening and questioning.

Mette's and Kari's contributions can be seen as if they brought two different texts into the field. Mette's notes showed the solution as $f(x) = 5 \cdot 2^x$. Kari showed her result as $f(x) = 5 \cdot 4^x$ and argues that it had to be correct. They investigated the calculations done. It seemed that each managed to follow the other's reasoning, but neither understood why the answers differed. Kari seemed convinced of her own solution, and Mette told that she agreed. In this respect Kari's model seemed unproblematic to them. Accordingly they focused on Mette's solution $f(x) = 5 \cdot 2^x$. They told that they could not understand why it was wrong. The calculations done by Mette seemed reasonable to them.

The students tried to understand the differences by investigating the two solutions in light of the other. It is my interpretation that the insight they had developed, and the insights they were trying to get about the differences, provoked them to continue (Renshaw & Brown 1999). *"I have used a month as the variable, Kari has used a quarter of a year. Is it possible that both solutions are correct? Could it be two ways of describing the same development?"* Mette asked. By doing some calculations, however, they showed to be convinced: $f(x) = 5 \cdot 4^x$ had to be correct and $f(x) = 5 \cdot 2^x$ could not be. Kari seemed satisfied; they had found the answer. Mette, however, was not comfortable. She was convinced that the solution $f(x) = 5 \cdot 4^x$ was correct, but she communicated that she could not understand why $f(x) = 5 \cdot 2^x$ was not.

I see this interaction as a conflict between the students' understandings; they develop different understanding in interaction with each other. It could be seen as comprising personal contributions and one could reflect on the process provoked by conflicts between personal perspectives. However, it seems more appropriate to focus on the different ways of understanding that are brought forth. Analyses on how the students

investigated the two models make me argue that they to a minor degree are defending each individual model. It appears as conflicts between understandings more than conflicts between the two students' *personal* understandings. The students positioned the different understandings in opposition to each other. They investigated the differences by seeing one in light of the other. The analyses developed in the study argue that Mette's main goal became to understand why $f(x) = 5 \cdot 2^x$ was not correct (and why the other one is correct). It is in this context when Mette turns to Kari and points at her notes:

"But if we think quarter, why is it $f(x) = 5 \cdot 4^x$? I cannot understand why it has to be $f(x) = 5 \cdot 4^x$!" Kari writes: $5 + 5 \cdot 3$. She talks when she writes: "*They were five and by the end of a period of one quarter each of them has recruited three.*" "Yes," Mette says, "*it corresponds with my thinking.*" (She points at some notes in her book.) Kari writes: $5+5 \cdot 3 = 5+5+5+5 = 5 \cdot 4$ "*This makes five plus five plus five plus five; that means five multiplied by four.....raised to the first, it makes five multiplied by four to the xth.*"¹ Mette looks at Kari and says "Yes...." It is obvious that she does not accept this as an explanation. "*I see that it is correct, but....*" They look at each other. They both seem confused. I interrupt by commenting: "*You see that it is correct, but you do not understand why it is?*" Mette looks gratefully at me.

I see this as representing, in several ways, conflicts between the arguments for the different solutions that was articulated:

- a) They got two *different answers*, and tried to find the correct one or to document how both of them were correct.
- b) They were searching for *different kinds of understanding* underpinning the solutions. It is obvious that Kari does not understand what Mette is searching for, neither is it easy for Mette to explain her intention. Kari commented on this in the following way: "*This happens over and over again. We do not listen well enough. I did not understand what she did not understand. I wanted her to understand it in my way!*" (Johnsen Høines 2002: 146)
- c) The two students *expressed themselves differently*. Kari's expressions can be characterised as a 'linear' text. The utterances were structured in a predictable way, as "we have this – this follows this, then this, and therefore this...." This can be seen in her writing and also in the oral language. The voice was distinct. The written symbolization seemed prior to the oral expressions. (Pimm 1987: 20). Kari repeated herself, she used the same words over and over again, she told how to do it in a

¹ In this paper I use parts of the subscript in order to underline some aspects of the research. Other parts, as for instance 'why four to the xth?' were important in the study, but not emphasised here.

rhythmic way. It is as if she let her oral language follow the written symbols. As she moved on, she tried to be more distinct, clearer. She tried to get Mette to understand.

Bakhtin (1981,1998) shows the importance of identifying the voices of the traditions or of the culture. He claims that the words are not just ours, we cannot use them freely. The voices from earlier users are implied within the words. This is realised in the interpretation. When listening to Kari's explanations, I could identify what I characterise as the teacher's voice, telling how to do or how to think. It is likely that Mette identified the teacher's voice, and that even Kari did so. Kari moves within a genre that is constituted by certain ways of expressing. The way of thinking is restricted and is given possibilities within the genre. The argumentation and solution are to be interpreted in the context of the genre.

Mette's way of expressing herself was not as easy follow. She talked slowly, she sounded inquiring, moving, comparing. Late in the situation she made a drawing:



to

Thoughtful and tarrying she said: "Oh.. yes..one..becomes.. four..that's ..why..it.. has.. to.. be.. four.. times.. (long pause)..yes, I understand." Mette organised the use of language in a different way. Accordingly she has got other limitations and other potentialities. The two texts appear as different due to the different genre within which they are expressed.

The situation can be described by how the two texts interact with each other. They are different and it becomes clear to me that the appearance of the other is a vital condition for the development.

AUTHORITATIVE TEXT

The utterances of Kari did not necessarily express the process in which Kari came to understand the relationships on which she is focusing. It was however the explanation she made in order to get Mette to understand. I found the teacher's voice a vital part of Kari's text. I have characterised it as linear, algebraic, rhythmic and repetitious. It showed an authoritative and monological tendency. It demanded "to do like this", or "this is how to understand". It did not invite for discussions. (Wertsch 1991)

I characterise Mette's text differently. It was as if she turned to and fro, searching. The utterance did not show the result of her thinking. The drawing was made slowly and investigative. It seemed to be a part of the process of getting insight. Mette's voice was a silent voice, though persisting. The way Mette structured her investigation is not easily grasped. Her process was not predictable.

Mette initiates a communicative turn when she after having got a new repetition from Kari, questioned the explanation. With energy in her voice, she attacked: "Then it

could be $4 \cdot 5$ as well, and then $4 \cdot 5^x$ " Kari looked at her, paralyzed: "Yes..... but..." They stopped, looking at each other, silent. I intervened: "Can you explain what you do not understand?" Mette, without hesitation: "That it is four times, that it is four times every time."

It appeared to me that Mette had a competent objection. She showed insight into the logical structure presented by Mette. She identified a logical gap. When she attacked the logical structure she also attacked the way of explaining, she attacked the teacher's voice. Kari was not able to answer. In the interview she confirmed that she did not have any alternative explanations. She had not seen the logical gap Mette had pointed to. Kari was limited within the frame of ways of express herself. Kari did not expect Mette's question; it was not invited for. They moved within different limitations.

A TEXT WITHIN THE TEXT

Looking into how the texts were contradicting with each other, particularly in the context of Mette's attack, made me identify another text within Mette's text (Mette 1 and Mette 2.)

The image shows handwritten mathematical work. At the top, it says $5 + 5 \cdot \frac{3}{3} = 5 \cdot \frac{3}{3} + 5$. Below this, it shows $= 5 \left(\frac{3}{3} + 1 \right)$. A box is drawn around the expression $5 \cdot \frac{3}{3} + 5$. Below the box, it says $f(x) = 5 \cdot 2^x$.

Her first solution (Mette1) seems similar to Kari's when it comes to the written and oral structure. They seem close in terms of genre. It has the same weakness that she attacks in Kari's explanation. This argues for seeing the process as interaction between three texts, between Kari and Mette1 and Mette2. In the study I find it reasonable to assume that when inquiring and attacking Kari's text, Mette attacked parts of her own text (Mette1). The

analyses show the possibility that Mette's listening to Kari's explanations helps to make some connections to her own reasoning. The authoritative text within her own text might be powerful condition for investigating the explanations she is offered by Kari. By the insight she has into the differences, she knows what kind of understanding she is (and is not) heading for. Her knowledge about the authoritative texts might be a condition for attacking them.

CONCLUSION

Investigating the students struggle to know, by moving between knowing and knowledge, helped me to get insight into how understanding develops as interactions or conflicts between different understandings. Words as *struggle*, *conflict* and *attack* is in this paper used to describe the processes in which the students conduct different ways of expressing in opposition to one another. The differences in the students' expressions are seen in an individual, social and genre perspective. The focus has been to show the importance of that the order carries in itself content, there is content

embedded within the genre. I argue for seeing the learners struggle to know in context of this.

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