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ERRATA: SOME CONDITION ON A POISSON DISTRIBUTION SERIES TO BE IN SUBCLASSES OF UNIVALENT FUNCTIONS

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Abstract. The purpose of this note is to give some mistyping corrections for our published article in [1].

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Keywords: Errata; Poisson distribution series; Analytic functions; Hadamard product; Starlike function; Convex functions.

These errata give the following correct statements for the corresponding statements on the cited page of our published article [1].

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Theorem 15. *The sufficient condition for $\mathcal{N}(m,\mu,\lambda;z)$ to be in the class $\mathcal{L}(A,B,\theta;\alpha)$ is*

$$\mu\lambda m^3+(\mu-\lambda+5\mu\lambda)m^2+(2\mu-2\lambda+1+4\mu\lambda)m-e^{-m}+1\leq\frac{(B-A)(1-\alpha)\cos\theta}{1+|B|}.\quad(5)$$

Proof. Since

$$\mathcal{N}(m,\mu,\lambda;z)=z+\sum_{n=2}^{\infty}[1+(n-1)(\mu-\lambda+n\mu\lambda)]\frac{m^{n-1}}{(n-1)!}e^{-m}z^n,\quad(z\in\mathbb{U}).$$

By applying Lemma 1, we need to prove that

$$\sum_{n=2}^{\infty}n(1+|B|)\left|[1+(n-1)(\mu-\lambda+n\mu\lambda)]\frac{m^{n-1}}{(n-1)!}e^{-m}\right|\leq(B-A)(1-\alpha)\cos\theta.$$

Thus,

$$I_2 = \sum_{n=2}^{\infty} n(1+|B|)[1+(n-1)(\mu-\lambda+n\mu\lambda)]\frac{m^{n-1}}{(n-1)!}e^{-m}$$
$$= (1+|B|)e^{-m}\Big[\sum_{n=2}^{\infty}(1+2\mu-2\lambda+4\mu\lambda)(n-1)\frac{m^{n-1}}{(n-1)!} + \sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!}$$

$$+ \sum_{n=2}^{\infty}(\mu-\lambda+5\mu\lambda)(n-1)(n-2)\frac{m^{n-1}}{(n-1)!} + \sum_{n=3}^{\infty}\mu\lambda(n-1)(n-2)(n-3)\frac{m^{n-1}}{(n-1)!}\Big]$$
$$= (1+|B|)e^{-m}\Big[\sum_{n=2}^{\infty}(1+2\mu-2\lambda+4\mu\lambda)m\frac{m^{n-2}}{(n-2)!} + \sum_{n=2}^{\infty}\frac{m^{n-1}}{(n-1)!} + \sum_{n=3}^{\infty}(\mu-\lambda+5\mu\lambda)m^2\frac{m^{n-3}}{(n-3)!}$$
$$+ \sum_{n=2}^{\infty}\mu\lambda m^3\frac{m^{n-4}}{(n-4)!}\Big]$$

$$= (1+|B|)e^{-m}[(1+2\mu-2\lambda+4\mu\lambda)m\sum_{n=0}^{\infty}\frac{m^n}{n!} + \sum_{n=1}^{\infty}\frac{m^n}{n!} + (\mu-\lambda+5\mu\lambda)m^2\sum_{n=0}^{\infty}\frac{m^n}{n!}$$
$$+ \mu\lambda m^3\sum_{n=0}^{\infty}\frac{m^n}{n!}]$$
$$= (1+|B|)e^{-m}[m(1+2\mu-2\lambda+4\mu\lambda)\sum_{n=0}^{\infty}\frac{m^n}{n!} + \sum_{n=0}^{\infty}\frac{m^n}{n!} - 1$$
$$+ (\mu-\lambda+5\mu\lambda)m^2\sum_{n=0}^{\infty}\frac{m^n}{n!} + \mu\lambda m^3\sum_{n=0}^{\infty}\frac{m^n}{n!}]$$
$$= (1+|B|)[\mu\lambda m^3 + (\mu-\lambda+5\mu\lambda)m^2 + (1+2\mu-2\lambda+4\mu\lambda)m + 1 - e^{-m}].$$

But this last equation is bounded by $(B-A)(1-\alpha)\cos\theta$ if Eq. (5) is holds. This completes the prove of Theorem 15.

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Corollary 16. *Let $A=-1$ and $B=1$ in Theorem 15, then the sufficient condition for $\mathcal{N}(m,\mu,\lambda;z)$ to be in the class $\mathcal{L}(\theta;\alpha)$ is*

$$\mu\lambda m^3 + (\mu-\lambda+5\mu\lambda)m^2 + (2\mu-2\lambda+1+4\mu\lambda)m - e^{-m} + 1 \le (1-\alpha)\cos\theta.$$

Corollary 17. *Let $\alpha = 0$ in Theorem 15, then the sufficient condition for $\mathcal{N}(m, \mu, \lambda; z)$ to be in the class $\mathcal{L}(A, B, \theta)$ is*

$$\mu\lambda m^3 + (\mu - \lambda + 5\mu\lambda)m^2 + (2\mu - 2\lambda + 1 + 4\mu\lambda)m - e^{-m} + 1 \leq \frac{(B - A)\cos\theta}{1 + |B|}.$$

Corollary 18. *Let $A = -\beta$, $B = \beta$, $\theta = 0$ and $\alpha = 0$ in Theorem 15, then the sufficient condition for $\mathcal{N}(m, \mu, \lambda; z)$ to be in the class $\mathcal{L}(-\beta, \beta, 0; 0) = \mathcal{D}(\beta)$ is*

$$\mu\lambda m^3 + (\mu - \lambda + 5\mu\lambda)m^2 + (2\mu - 2\lambda + 1 + 4\mu\lambda)m - e^{-m} + 1 \leq \frac{2\beta}{1 + |\beta|}.$$

Corollary 19. *Let $A = -\beta$, $B = \beta$ and $\theta = 0$ in Theorem 15, then the sufficient condition for $\mathcal{N}(m, \mu, \lambda; z)$ to be in the class $\mathcal{R}(\beta; \alpha)$ is*

$$\mu\lambda m^3 + (\mu - \lambda + 5\mu\lambda)m^2 + (2\mu - 2\lambda + 1 + 4\mu\lambda)m - e^{-m} + 1 \leq \frac{2\beta(1 - \alpha)}{1 + |\beta|}.$$

References

[1] R. M. El-Ashwah and W. Y. Kota, Some Condition on a Poisson Distribution Series to be in Subclasses of Univalent Functions, Acta Universitatis Apulensis, 51 (2017), 89-103.

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