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NEW CLASSES CONTAINING GENERALIZED SĂLĂGEAN OPERATOR AND RUSCHEWEYH DERIVATIVE

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Abstract. In this paper we introduce new classes containig the linear operator $RD^n_{\lambda,\alpha} : \mathcal{A} \to \mathcal{A}$, $RD^n_{\lambda,\alpha}f(z) = (1-\alpha)R^n f(z) + \alpha D^n_\lambda f(z)$, $z \in U$, where $R^n f(z)$ is the Ruscheweyh derivative, $D^n_\lambda f(z)$ the generalized Sălăgean operator and $\mathcal{A}_n = \{f \in \mathcal{H}(U) : f(z) = z + a_{n+1}z^{n+1} + \dots,\ z \in U\}$ is the class of normalized analytic functions with $\mathcal{A}_1 = \mathcal{A}$. Characterization and other properties of these classes are studied.

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1. Introduction

Denote by U the unit disc of the complex plane, $U = \{z \in \mathbb{C} : |z| < 1\}$ and $\mathcal{H}(U)$ the space of holomorphic functions in U.

Let $\mathcal{A}_n = \{f \in \mathcal{H}(U) : f(z) = z + a_{n+1}z^{n+1} + \dots,\ z \in U\}$ with $\mathcal{A}_1 = \mathcal{A}$.

Definition 1. *(Al Oboudi [8]) For $f \in \mathcal{A}$, $\lambda \geq 0$ and $n \in \mathbb{N}$, the operator D^n_λ is defined by $D^n_\lambda : \mathcal{A} \to \mathcal{A}$,*

$$\begin{aligned} D^0_\lambda f(z) &= f(z) \\ D^1_\lambda f(z) &= (1-\lambda) f(z) + \lambda z f'(z) = D_\lambda f(z), \ \dots \\ D^{n+1}_\lambda f(z) &= (1-\lambda) D^n_\lambda f(z) + \lambda z \left(D^n_\lambda f(z)\right)' = D_\lambda \left(D^n_\lambda f(z)\right), \quad z \in U. \end{aligned}$$

Remark 1. *If $f \in \mathcal{A}$ and $f(z) = z + \sum_{j=2}^{\infty} a_j z^j$, then $D^n_\lambda f(z) = z + \sum_{j=2}^{\infty} \left[1 + (j-1)\lambda\right]^n a_j z^j$, $z \in U$.*

Remark 2. *For $\lambda = 1$ in the above definition we obtain the Sălăgean differential operator [13].*

Definition 2. *(Ruscheweyh [12]) For $f \in \mathcal{A}$, $n \in \mathbb{N}$, the operator R^n is defined by $R^n : \mathcal{A} \to \mathcal{A}$,*

$$\begin{aligned} R^0 f(z) &= f(z) \\ R^1 f(z) &= z f'(z), \ ... \\ (n+1) R^{n+1} f(z) &= z (R^n f(z))' + n R^n f(z), \quad z \in U. \end{aligned}$$

Remark 3. *If $f \in \mathcal{A}$, $f(z) = z + \sum_{j=2}^{\infty} a_j z^j$, then $R^n f(z) = z + \sum_{j=2}^{\infty} \frac{(n+j-1)!}{n!(j-1)!} a_j z^j$, $z \in U$.*

Definition 3. *[3] Let $\gamma, \alpha \geq 0$, $n \in \mathbb{N}$. Denote by $RD_{\lambda,\alpha}^n$ the operator given by $RD_{\lambda,\alpha}^n : \mathcal{A} \to \mathcal{A}$,*

$$RD_{\lambda,\alpha}^n f(z) = (1-\alpha) R^n f(z) + \alpha D_{\lambda}^n f(z), \qquad z \in U.$$

Remark 4. *If $f \in \mathcal{A}$, $f(z) = z + \sum_{j=2}^{\infty} a_j z^j$, then*
$RD_{\lambda,\alpha}^n f(z) = z + \sum_{j=2}^{\infty} \left\{ \alpha \left[1 + (j-1)\lambda\right]^n + (1-\alpha) \frac{(n+j-1)!}{n!(j-1)!} \right\} a_j z^j$, $z \in U$.

This operator was studied also in [4], [6], [7], [9], [10].

Remark 5. *For $\gamma = 0$, $RD_{\lambda,0}^n f(z) = R^n f(z)$, where $z \in U$ and for $\gamma = 1$, $RD_{\lambda,1}^n f(z) = D_{\lambda}^n f(z)$, where $z \in U$.*

For $\lambda = 1$, we obtain $RD_{1,\alpha}^n f(z) = L_{\alpha}^n f(z)$ which was studied in [1], [2] and [5].

Definition 4. *Let $f \in \mathcal{A}$. Then $f(z)$ is in the class $\mathcal{S}_{\lambda,\alpha}^n(\mu)$ if and only if*

$$Re \left(\frac{z \left(RD_{\lambda,\alpha}^n f(z) \right)'}{RD_{\lambda,\alpha}^n f(z)} \right) > \mu, \ \ 0 \leq \mu < 1, \ \ z \in U.$$

Definition 5. *Let $f \in \mathcal{A}$. Then $f(z)$ is in the class $\mathcal{C}_{\lambda,\alpha}^n(\mu)$ if and only if*

$$Re \left(\frac{\left[z \left(RD_{\lambda,\alpha}^n f(z) \right)' \right]'}{\left(RD_{\lambda,\alpha}^n f(z) \right)'} \right) > \mu, \ \ 0 \leq \mu < 1, \ \ z \in U.$$

We study the charaterization and distortion theorems, and other properties of these classes, following the paper of M. Darus and R. Ibrahim [11].

2. General properties of $RD^n_{\lambda,\alpha}$

In this section we study the characterization properties and distortion theorems for the function $f(z) \in \mathcal{A}$ to belong to the classes $\mathcal{S}^n_{\lambda,\alpha}(\mu)$ and $\mathcal{C}^n_{\lambda,\alpha}(\mu)$ by obtaining the coefficient bounds.

Theorem 1. *Let $f \in \mathcal{A}$. If*

$$\sum_{j=2}^{\infty}(j-\mu)\left\{\alpha\left[1+(j-1)\lambda\right]^n+(1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j| \le 1-\mu, \quad 0\le\mu<1, \tag{1}$$

then $f(z) \in \mathcal{S}^n_{\lambda,\alpha}(\mu)$. The result (1) is sharp.

Proof. Suppose that (1) holds. Since

$$1-\mu \ge \sum_{j=2}^{\infty}(j-\mu)\left\{\alpha\left[1+(j-1)\lambda\right]^n+(1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j|$$
$$\ge \mu\sum_{j=2}^{\infty}\left\{\alpha\left[1+(j-1)\lambda\right]^n+(1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j| -$$
$$\sum_{j=2}^{\infty}j\left\{\alpha\left[1+(j-1)\lambda\right]^n+(1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j|$$

then this implies that

$$\frac{1+\sum_{j=2}^{\infty}j\left\{\alpha\left[1+(j-1)\lambda\right]^n+(1-\alpha)\frac{(n+j-1)!}{n!(j-1)!}\right\}|a_j|}{1+\sum_{j=2}^{\infty}\left\{\alpha\left[1+(j-1)\lambda\right]^n+(1-\alpha)\frac{(n+j-1)!}{n!(j-1)!}\right\}|a_j|} > \mu.$$

So, we deduce that

$$Re\left(\frac{z\left(RD^n_{\lambda,\alpha}f(z)\right)'}{RD^n_{\lambda,\alpha}f(z)}\right) > \mu,\ \ 0\le\mu<1,\ \ z\in U.$$

We have $f(z) \in \mathcal{S}^n_{\lambda,\alpha}(\mu)$, which evidently completes the proof.

The assertion (1) is sharp and the extremal function is given by

$$f(z) = z+\sum_{j=2}^{\infty}\frac{(1-\mu)}{(j-\mu)\left\{\alpha\left[1+(j-1)\lambda\right]^n+(1-\alpha)\frac{(n+j-1)!}{n!(j-1)!}\right\}}z^j.$$

Corollary 2. *Let the hypotheses of Theorem 1 satisfy. Then*

$$|a_j| \leq \frac{1-\mu}{(j-\mu)\left\{\alpha\left[1+(j-1)\lambda\right]^n+(1-\alpha)\frac{(n+j-1)!}{n!(j-1)!}\right\}}, \ \ \forall\, j\geq 2.$$

Theorem 3. *Let $f\in\mathcal{A}$. If*

$$\sum_{j=2}^{\infty} j(j-\mu)\left\{\alpha\left[1+(j-1)\lambda\right]^n+(1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j| \leq 1-\mu, \quad 0\leq\mu<1, \tag{2}$$

then $f(z)\in\mathcal{C}^n_{\lambda,\alpha}(\mu)$. The result (2) is sharp.

Proof. Suppose that (2) holds. Since

$$\begin{aligned} 1-\mu \ &\geq \ \sum_{j=2}^{\infty} j(j-\mu)\left\{\alpha\left[1+(j-1)\lambda\right]^n+(1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j| \\ &\geq \ \mu\sum_{j=2}^{\infty} j\left\{\alpha\left[1+(j-1)\lambda\right]^n+(1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j| - \\ &\quad\ \sum_{j=2}^{\infty} j^2\left\{\alpha\left[1+(j-1)\lambda\right]^n+(1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j| \end{aligned}$$

then this implies that

$$\frac{1+\sum_{j=2}^{\infty} j^2\left\{\alpha\left[1+(j-1)\lambda\right]^n+(1-\alpha)\frac{(n+j-1)!}{n!(j-1)!}\right\}|a_j|}{1+\sum_{j=2}^{\infty} j\left\{\alpha\left[1+(j-1)\lambda\right]^n+(1-\alpha)\frac{(n+j-1)!}{n!(j-1)!}\right\}|a_j|} > \mu.$$

So, we deduce that

$$Re\left(\frac{\left[z\left(RD^n_{\lambda,\alpha}f(z)\right)'\right]'}{\left(RD^n_{\lambda,\alpha}f(z)\right)'}\right) > \mu, \ \ 0\leq\mu<1, \ \ z\in U.$$

We have $f(z)\in\mathcal{C}^n_{\lambda,\alpha}(\mu)$, which evidently completes the proof.

The assertion (2) is sharp and the extremal function is given by

$$f(z) = z+\sum_{j=2}^{\infty}\frac{(1-\mu)}{j(j-\mu)\left\{\alpha\left[1+(j-1)\lambda\right]^n+(1-\alpha)\frac{(n+j-1)!}{n!(j-1)!}\right\}}z^j.$$

Corollary 4. *Let the hypotheses of Theorem 3 be satisfied. Then*

$$|a_j| \leq \frac{1-\mu}{j\,(j-\mu)\left\{\alpha\,[1+(j-1)\,\lambda]^n+(1-\alpha)\,\frac{(n+j-1)!}{n!(j-1)!}\right\}},\quad \forall\ j\geq 2.$$

Also, we have the following inclusion results:

Theorem 5. *Let* $0 \leq \mu_1 \leq \mu_2 < 1$. *Then* $\mathcal{S}^n_{\lambda,\alpha}(\mu_1) \supseteq \mathcal{S}^n_{\lambda,\alpha}(\mu_2)$.

Proof. By Theorem 1.

Theorem 6. *Let* $0 \leq \mu_1 \leq \mu_2 < 1$. *Then* $\mathcal{C}^n_{\lambda,\alpha}(\mu_1) \supseteq \mathcal{C}^n_{\lambda,\alpha}(\mu_2)$.

Proof. By Theorem 3.

We introduce the following distortion theorems.

Theorem 7. *Let the function* $f \in \mathcal{A}$ *and*

$$\sum_{j=2}^{\infty}(j-\mu)\left\{\alpha\,[1+(j-1)\,\lambda]^n+(1-\alpha)\,\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j| \leq 1-\mu,\quad 0\leq\mu<1.$$

Then for $z \in U$ *and* $0 \leq \mu < 1$,

$$\left|RD^n_{\lambda,\alpha}f(z)\right| \geq |z| - \frac{1-\mu}{2-\mu}\,|z|^2$$

and

$$\left|RD^n_{\lambda,\alpha}f(z)\right| \leq |z| + \frac{1-\mu}{2-\mu}\,|z|^2.$$

Proof. By using Theorem 1, one can verify that

$$(2-\mu)\sum_{j=2}^{\infty}\left\{\alpha\,[1+(j-1)\,\lambda]^n+(1-\alpha)\,\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j| \leq$$
$$\sum_{j=2}^{\infty}(j-\mu)\left\{\alpha\,[1+(j-1)\,\lambda]^n+(1-\alpha)\,\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j| \leq 1-\mu.$$

Hence,

$$\sum_{j=2}^{\infty}\left\{\alpha\,[1+(j-1)\,\lambda]^n+(1-\alpha)\,\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j| \leq \frac{1-\mu}{2-\mu}.$$

We obtain

$$\left|RD^n_{\lambda,\alpha}f(z)\right| = \left|z + \sum_{j=2}^{\infty}\left\{\alpha\left[1+(j-1)\lambda\right]^n + (1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}a_j z^j\right| \leq$$

$$|z| + \sum_{j=2}^{\infty}\left\{\alpha\left[1+(j-1)\lambda\right]^n + (1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j|\,|z|^j \leq$$

$$|z| + \sum_{j=2}^{\infty}\left\{\alpha\left[1+(j-1)\lambda\right]^n + (1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j|\,|z|^2 \leq |z| + \frac{1-\mu}{2-\mu}|z|^2.$$

The other assertion can be proved as follows

$$\left|RD^n_{\lambda,\alpha}f(z)\right| = \left|z + \sum_{j=2}^{\infty}\left\{\alpha\left[1+(j-1)\lambda\right]^n + (1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}a_j z^j\right| \geq$$

$$|z| - \sum_{j=2}^{\infty}\left\{\alpha\left[1+(j-1)\lambda\right]^n + (1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j|\,|z|^j \geq$$

$$|z| - \sum_{j=2}^{\infty}\left\{\alpha\left[1+(j-1)\lambda\right]^n + (1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j|\,|z|^2 \geq |z| - \frac{1-\mu}{2-\mu}|z|^2.$$

This completes the proof.

Theorem 8. *Let the function $f \in \mathcal{A}$ and*

$$\sum_{j=2}^{\infty} j(j-\mu)\left\{\alpha\left[1+(j-1)\lambda\right]^n + (1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j| \leq 1-\mu, \quad 0 \leq \mu < 1.$$

Then for $z \in U$ and $0 \leq \mu < 1$,

$$\left|RD^n_{\lambda,\alpha}f(z)\right| \geq |z| - \frac{1-\mu}{2\,(2-\mu)}|z|^2$$

and

$$\left|RD^n_{\lambda,\alpha}f(z)\right| \leq |z| + \frac{1-\mu}{2\,(2-\mu)}|z|^2.$$

Proof. By using Theorem 3, one can verify that

$$2\,(2-\mu)\sum_{j=2}^{\infty}\left\{\alpha\left[1+(j-1)\lambda\right]^n + (1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j| \leq$$

$$\sum_{j=2}^{\infty} j(j-\mu)\left\{\alpha\left[1+(j-1)\lambda\right]^n + (1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j| \leq 1-\mu.$$

Hence,

$$\sum_{j=2}^{\infty}\left\{\alpha\left[1+(j-1)\lambda\right]^{n}+(1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j|\leq\frac{1-\mu}{2\,(2-\mu)}.$$

We obtain

$$\left|RD^{n}_{\lambda,\alpha}f(z)\right|=\left|z+\sum_{j=2}^{\infty}\left\{\alpha\left[1+(j-1)\lambda\right]^{n}+(1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}a_jz^j\right|\leq$$

$$|z|+\sum_{j=2}^{\infty}\left\{\alpha\left[1+(j-1)\lambda\right]^{n}+(1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j|\,|z|^j\leq$$

$$|z|+\sum_{j=2}^{\infty}\left\{\alpha\left[1+(j-1)\lambda\right]^{n}+(1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j|\,|z|^2\leq|z|+\frac{1-\mu}{2\,(2-\mu)}\,|z|^2.$$

The other assertion can be proved as follows

$$\left|RD^{n}_{\lambda,\alpha}f(z)\right|=\left|z+\sum_{j=2}^{\infty}\left\{\alpha\left[1+(j-1)\lambda\right]^{n}+(1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}a_jz^j\right|\geq$$

$$|z|-\sum_{j=2}^{\infty}\left\{\alpha\left[1+(j-1)\lambda\right]^{n}+(1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j|\,|z|^j\geq$$

$$|z|-\sum_{j=2}^{\infty}\left\{\alpha\left[1+(j-1)\lambda\right]^{n}+(1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j|\,|z|^2\geq|z|-\frac{1-\mu}{2\,(2-\mu)}\,|z|^2.$$

This completes the proof.

Also, we have the following distortion results.

Theorem 9. *Let the hypotheses of Theorem 1 be satisfied. Then*

$$|f(z)|\geq|z|-\frac{1-\mu}{(2-\mu)\left[\alpha\,(1+\lambda)^{n}+(1-\alpha)\,(n+1)\right]}\,|z|^2$$

and

$$|f(z)|\leq|z|+\frac{1-\mu}{(2-\mu)\left[\alpha\,(1+\lambda)^{n}+(1-\alpha)\,(n+1)\right]}\,|z|^2.$$

Proof. In virtue of Theorem 1, we have

$$(2-\mu)\left[\alpha(1+\lambda)^n+(1-\alpha)(n+1)\right]\sum_{j=2}^{\infty}|a_j| \leq$$

$$\sum_{j=2}^{\infty}(j-\mu)\left\{\alpha\left[1+(j-1)\lambda\right]^n+(1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j| \leq 1-\mu,$$

thus,

$$\sum_{j=2}^{\infty}|a_j| \leq \frac{1-\mu}{(2-\mu)\left[\alpha(1+\lambda)^n+(1-\alpha)(n+1)\right]}.$$

We obtain

$$|f(z)| = \left|z+\sum_{j=2}^{\infty}a_j z^j\right| \leq |z|+\sum_{j=2}^{\infty}|a_j|\,|z|^2 \leq$$

$$|z|+\frac{1-\mu}{(2-\mu)\left[\alpha(1+\lambda)^n+(1-\alpha)(n+1)\right]}|z|^2.$$

The other assertion can be proved as follows:

$$|f(z)| \geq |z|-\sum_{j=2}^{\infty}|a_j|\,|z|^2 \geq |z|-\frac{1-\mu}{(2-\mu)\left[\alpha(1+\lambda)^n+(1-\alpha)(n+1)\right]}|z|^2.$$

This completes the proof.

In the same way we can get the following result.

Theorem 10. *Let the hypotheses of Theorem 3 be satisfied. Then* $(j-\mu)\left\{\alpha\left[1+(j-1)\lambda\right]^n+(1-\alpha)\frac{(n+j-1)!}{n!(j-1)!}\right\} \geq 0$ *and* $0 \leq \mu < 1$ *poses*

$$|f(z)| \geq |z|-\frac{1-\mu}{2(2-\mu)\left[\alpha(1+\lambda)^n+(1-\alpha)(n+1)\right]}|z|^2$$

and

$$|f(z)| \leq |z|+\frac{1-\mu}{2(2-\mu)\left[\alpha(1+\lambda)^n+(1-\alpha)(n+1)\right]}|z|^2.$$

Proof. In virtue of Theorem 3, we have

$$2(2-\mu)\left[\alpha(1+\lambda)^n+(1-\alpha)(n+1)\right]\sum_{j=2}^{\infty}|a_j| \leq$$

$$\sum_{j=2}^{\infty}j(j-\mu)\left\{\alpha\left[1+(j-1)\lambda\right]^n+(1-\alpha)\frac{(n+j-1)!}{n!\,(j-1)!}\right\}|a_j| \leq 1-\mu,$$

thus,

$$\sum_{j=2}^{\infty} |a_j| \leq \frac{1-\mu}{2(2-\mu)\left[\alpha(1+\lambda)^n + (1-\alpha)(n+1)\right]}.$$

We obtain

$$|f(z)| = \left| z + \sum_{j=2}^{\infty} a_j z^j \right| \leq |z| + \sum_{j=2}^{\infty} |a_j| \, |z|^2 \leq$$

$$|z| + \frac{1-\mu}{2(2-\mu)\left[\alpha(1+\lambda)^n + (1-\alpha)(n+1)\right]} |z|^2 .$$

The other assertion can be proved as follows:

$$/ \, |f(z)| = \left| z + \sum_{j=2}^{\infty} a_j z^j \right| \geq |z| - \sum_{j=2}^{\infty} |a_j| \, |z|^2 \geq$$

$$|z| - \frac{1-\mu}{2(2-\mu)\left[\alpha(1+\lambda)^n + (1-\alpha)(n+1)\right]} |z|^2 .$$

This completes the proof.

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