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CERTAIN NEW SUBCLASS OF CLOSE-TO-CONVEX FUNCTIONS

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Abstract. In this work, we introduce and investigate two new subclass of analytic and close-to-convex functions in the open unit disk $\mathbb{U}$. For each of these function classes, several coefficient inequalities are established. The usefulness of the main results are depicted by showing improvement in earlier results.

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1. Introduction

Let $\mathcal{H}$ denote the class of analytic functions in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$ and $\mathcal{A}$ denote class of functions $f \in \mathcal{H}$ normalized by $f(0) = 0, f'(0) = 1$. A function $f \in \mathcal{A}$ has series representation of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad z \in \mathbb{U}. \tag{1}$$

We denote by $\mathcal{S}$ the subclass $\mathcal{A}$ consisting of all functions in $\mathcal{A}$, which are also univalent in $\mathbb{U}$. Let $\Omega = \{w \in \mathcal{H} : \ w(0) = 0, \ |w(z)| < 1\}$. We say that $f \in \mathcal{H}$ is subordinate to $g \in \mathcal{H}$ in the unit disk $\mathbb{U}$, written $f \prec g$, if there exists a function $w \in \Omega$ such that $f(z) = g(w(z))$ for $z \in \mathbb{U}$. Therefore $f \prec g$ in $\mathbb{U}$ implies $f(\mathbb{U}) \subset g(\mathbb{U})$. Furthermore, if the function g is univalent in U, then $f \prec g$, if and only if, $f(0) = g(0)$ and $f(\mathbb{U}) \subset g(\mathbb{U})$ in $\mathbb{U}$.

A function $f \in \mathcal{A}$ is said to belongs to the class $\mathcal{S}^*(\alpha)$ of *starlike function of order* α in $\mathbb{U}$, if it satisfies the inequality

$$\Re\left(\frac{zf'(z)}{f(z)}\right) > \alpha \quad (z \in \mathbb{U}, 0 \leq \alpha < 1).$$

Further, a function f is said to belongs to the class $\mathcal{K}(\alpha)$ of *close-to-convex function of order* α in $\mathbb{U}$, if $g \in \mathcal{S}^*(\alpha)$ and satisfies the following inequality

$$\Re\left(\frac{zf'(z)}{g(z)}\right) > \alpha \quad (z \in \mathbb{U}, 0 \leq \alpha < 1).$$

Obviously $\mathcal{S}^*(0) = \mathcal{S}^*$ and $\mathcal{K}(0) = \mathcal{K}$.

Gao and Zhou [1] studied a subclass $\mathcal{K}_s$ of analytic functions, that is, a function $f \in \mathcal{S}$ said to be in the class $\mathcal{K}_s$, if it satisfies the inequality:

$$\Re\left(\frac{z^2 f'(z)}{g(z)g(-z)}\right) < 0, \quad z \in \mathbb{U},$$

where $g \in \mathcal{S}^*(1/2)$. Recently, Kowalczyk and Les-Bomba [5] generalize the class $\mathcal{K}_s$, that is, a function $f \in \mathcal{S}$ said to be in the class $\mathcal{K}_s(\gamma)$, if it satisfies the inequality:

$$\Re\left(\frac{-z^2 f'(z)}{g(z)g(-z)}\right) > \gamma, \quad z \in \mathbb{U},$$

where $0 \leq \gamma < 1$ and $g \in \mathcal{S}^*(1/2)$. Note that $\mathcal{K}_s(0) \equiv \mathcal{K}_s$. More recently, Seker [8] further generalize the class $\mathcal{K}_s(\gamma)$ with respect to k-symmetric points, that is, a function $f \in \mathcal{S}$ said to be in the class $\mathcal{K}_s^{(k)}(\gamma)$, if it satisfies the inequality:

$$\Re\left(\frac{z^k f'(z)}{g_k(z)}\right) > \gamma, \quad z \in \mathbb{U},$$

where $0 \leq \gamma < 1$ and $g \in \mathcal{S}^*(\frac{k-1}{k})$, $k \geq 1$, is a fixed positive integer and $g_k(z)$ defined by the following inequality

$$g_k(z) = \prod_{i=0}^{i-1} \varepsilon^{-i} g(\varepsilon^i z), \qquad \varepsilon^k = 1.$$

Note that $\mathcal{K}_s^{(2)}(\gamma) = \mathcal{K}_s(\gamma)$ and $\mathcal{K}_s^{(2)}(0) = \mathcal{K}_s$. Several other subclasses of *close-to-convex functions* have been studied recently. We choose recall here the investigation by (for example) Goyal and Goswami [2], Wang *et al.* [9], Xu *al.* [10].

Motivated by aforementioned work and a class of analytic functions studied by Owa *et al.* [7], we introduce a new class $\mathcal{X}_t(\gamma)$, which is subclass of the class $\mathcal{K}_s(\gamma)$.

Definition 1. *A function $f \in \mathcal{S}$ said to be in the class $\mathcal{X}_t(\gamma)$, if it satisfies the inequality:*

$$\Re\left(\frac{tz^2 f'(z)}{g(z)g(tz)}\right) > \gamma, \qquad z \in \mathbb{U}, \tag{2}$$

where $0 \leq \gamma < 1$, $|t| \leq 1, t \neq 0$ and $g \in \mathcal{S}^*(1/2)$.

A simple calculation shows that the inequality (1.2) is equivalent to

$$\left|\frac{tz^2 f'(z)}{g(z)g(tz)} - 1\right| < \left|\frac{tz^2 f'(z)}{g(z)g(tz)} + 1 - 2\gamma\right|, \quad z \in \mathbb{U}. \tag{3}$$

Note that, $\mathcal{X}_{-1}(\gamma) = \mathcal{K}_s(\gamma)$ and $\mathcal{X}_{-1}(0) = \mathcal{K}_s$. Further we observe that the class $\mathcal{K}_s^{(k)}(\gamma)$ studied by Seker [8] is different from the class $\mathcal{X}_t(\gamma)$. In the class $\mathcal{K}_s^{(k)}(\gamma)$, *Seker* consider k-symmetric points in the unit disk $\mathbb{U}$, whereas in the class $\mathcal{X}_t(\gamma)$, we consider arbitrary points in the unit disk $\mathbb{U}$. However, $\mathcal{K}_s^{(2)}(\gamma) \equiv \mathcal{X}_{-1}(\gamma)$.

Example. We observe that, the function

$$\mathcal{F}(z) = \frac{2\gamma - 1 - t}{(1-t)^2} \ln \frac{1 - tz}{1 - z} + \frac{2(1-\gamma)z}{(1-t)(1-z)}, \quad z \in \mathbb{U},$$

belongs to the class $\mathcal{X}_t(\gamma)$, where $0 \le \gamma < 1,\ |t| \le 1, t \ne 0$. Indeed, $\mathcal{F}$ is analytic in $\mathbb{U}$ and $f(0) = 0$. Moreover,

$$\mathcal{F}'(z) = \frac{1 + (1 - 2\gamma)z}{(1 - tz)(1 - z)^2}, \quad z \in \mathbb{U}.$$

If we put $g(z) = z/(1-z),\ z \in \mathbb{U}$, then it is clear that $g \in \mathcal{S}^*(1/2)$ and

$$\Re\left(\frac{tz^2\mathcal{F}'(z)}{g(z)g(tz)}\right) = \Re\left(\frac{1 + (1-2\gamma)z}{1 - z}\right) > \gamma, \ \ z \in \mathbb{U}.$$

This means that $\mathcal{F} \in \mathcal{X}_t(\gamma)$ and is generated by $g = z/(1-z)$.

Remark 1. *If* $g(z) \in \mathcal{A}$, *given by*

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n, \quad z \in \mathbb{U}, \tag{4}$$

then

$$F(z) = \frac{g(z)g(tz)}{tz} = z + \sum_{n=2}^{\infty} c_n z^n, \quad z \in \mathbb{U}, \tag{5}$$

where

$$c_n = b_n + b_2 b_{n-1} t + b_3 b_{n-2} t^2 + ... + b_{n-1} b_2 t^{n-2} + b_n t^{n-1}. \tag{6}$$

Further, if $g \in \mathcal{S}^*(1/2)$ *defined above by (4) and* $|tz| \le |z| < 1$, *then from the definitions of starlike function, we have*

$$\Re\left(\frac{zF'(z)}{F(z)}\right) = \Re\left(\frac{zg'(z)}{g(z)}\right) + \Re\left(\frac{tz\, g'(tz)}{g(tz)}\right) - 1 > 0.$$

Therefore, $F(z) = \dfrac{g(z)g(tz)}{tz} \in \mathcal{S}^*$ *and thus*

$$\mathcal{X}_t(\gamma) \ \subset \ \mathcal{K}_s(\gamma) \ \subset \ \mathcal{K}_s \ \subset \ \mathcal{K} \subset \ \mathcal{S}.$$

We further generalize class $\mathcal{X}_t(\gamma)$, as follows:

Definition 2. *a function $f \in \mathcal{S}$ said to be in the class $\mathcal{X}_t(h)$, if there exist a function $g \in \mathcal{S}^*(1/2)$ such that*

$$\frac{tz^2 f'(z)}{g(z)g(tz)} \in h(\mathbb{U}), \quad (z \in \mathbb{U},\, |t| \leq 1,\, t \neq 0), \tag{7}$$

where $h : \mathbb{U} \to \mathbb{C}$ be a complex function such that $h(0) = 1$ and $h(\overline{z}) = \overline{h(z)}$ $(z \in \mathbb{U};\, \Re(h(z)) > 0)$. Suppose also that the function h satisfies the following conditions:

$$\begin{cases} \min_{|z|=r} |h(z)| = \min\{h(r), h(-r)\}, \quad 0 < r < 1, \\ \\ \max_{|z|=r} |h(z)| = \max\{h(r), h(-r)\}, \quad 0 < r < 1. \end{cases} \tag{8}$$

Note that, $\mathcal{X}_{-1}(h) \equiv \mathcal{K}_s(h)$ is the class studied by Xu *et al.* [10].

Remark 2. *From the definition, it is clear that a function $f \in \mathcal{S}$ said to be in the class $\mathcal{X}_t(h)$, if there exist a function $g \in \mathcal{S}^*(1/2)$ such that*

$$\frac{zf'(z)}{F(z)} \in h(\mathbb{U}), \quad z \in \mathbb{U},$$

where $F(z)$ given by (5) *and member of the class of starlike functions. Therefore $f \in \mathcal{K}$. Thus we have $\mathcal{X}_t(h) \subset \mathcal{K} \subset \mathcal{S}$.*

In the present paper, we obtained certain coefficient inequalities of the classes $\mathcal{X}_t(\gamma)$ and $\mathcal{X}_t(h)$.

2. Main Results

We first prove the following result.

Theorem 1. *Let an analytic function $f(z)$ be given by* (1) *and $g \in \mathcal{S}^*(1/2)$ given by* (4)*. If $f \in \mathcal{X}_t(h)$, then*

$$1 + \sum_{n=2}^{\infty} \left(\frac{na_n - h(x)c_n}{1 - h(x)} \right) z^{n-1} \neq 0 \quad (|x| = 1; z \in \mathbb{U}). \tag{9}$$

Proof. Suppose that

$$\frac{tz^2 f'(z)}{g(z)g(tz)} \neq h(x) \quad (|x| = 1; z \in \mathbb{U})$$

or

$$f'(z) \neq h(x)\left(\frac{g(z)g(tz)}{tz^2}\right) \quad (|x| = 1; z \in \mathbb{U}).$$

Now using (1) and (5) and simplifying, we get the desired result (9).

On taking

$$h(x) = h(e^{i\theta}) = \frac{1 + (1-2\gamma)e^{i\theta}}{1 - e^{i\theta}} \quad (0 < \theta < 2\pi;\ 0 \leq \gamma < 1) \tag{10}$$

in Theorem 2.1, we obtain

Theorem 2. *Let an analytic function $f(z)$ be given by (1) and $g \in \mathcal{S}^*(1/2)$ given by (4). If $f \in \mathcal{X}_t(\gamma)$, then*

$$\sum_{n=2}^{\infty} \{2n\,|a_n| + (1 + |1-2\gamma|)\,|c_n|\} \leq 2(1-\gamma), \quad z \in \mathbb{U}, \tag{11}$$

where the coefficient of c_n $(n = 2, 3, \cdots)$ are given by (6).

Proof. If $f \in \mathcal{X}_t(\gamma)$, then (9) holds true. Using (10) in (9), we get

$$1 - \sum_{n=2}^{\infty} \frac{e^{-i\theta} - 1}{2(1-\gamma)}\, n a_n z^{n-1} + \sum_{n=2}^{\infty} \frac{e^{-i\theta} + 1 - 2\gamma}{2(1-\gamma)}\, c_n z^{n-1} \neq 0, \quad z \in \mathbb{U}.$$

Now

$$\left|1 - \sum_{n=2}^{\infty} \frac{e^{-i\theta} - 1}{2(1-\gamma)}\, n a_n z^{n-1} + \sum_{n=2}^{\infty} \frac{e^{-i\theta} + 1 - 2\gamma}{2(1-\gamma)}\, c_n z^{n-1}\right|$$

$$\begin{aligned}
&\geq 1 - \left|\sum_{n=2}^{\infty} \frac{e^{-i\theta} - 1}{2(1-\gamma)}\, n a_n z^{n-1} - \sum_{n=2}^{\infty} \frac{e^{-i\theta} + 1 - 2\gamma}{2(1-\gamma)}\, c_n z^{n-1}\right| \\
&\geq 1 - \left|\sum_{n=2}^{\infty} \frac{e^{-i\theta} - 1}{2(1-\gamma)}\, n a_n z^{n-1}\right| - \left|\sum_{n=2}^{\infty} \frac{e^{-i\theta} + 1 - 2\gamma}{2(1-\gamma)}\, c_n z^{n-1}\right| \\
&\geq 1 - \sum_{n=2}^{\infty} \frac{2}{2(1-\gamma)}\, n|a_n| - \sum_{n=2}^{\infty} \frac{1 + |1-2\gamma|}{2(1-\gamma)}\, |c_n| \geq 0
\end{aligned}$$

Which is equivalent to inequality (11). This complete the proof of Theorem 2.

Theorem 3. *Let an analytic function* $f(z)$ *be given by* (1) *and* $g \in \mathcal{S}^*(1/2)$ *given by* (4)*. If inequality* (11) *holds true, then* $f \in \mathcal{X}_t(\gamma)$*.*

Proof. For $z \in \mathbb{U}$, we have

$$\begin{aligned}
A &= \left| zf'(z) - \frac{g(z)g(tz)}{tz} \right| - \left| zf'(z) + (1-2\gamma)\frac{g(z)g(tz)}{tz} \right| \\
&= \left| \sum_{n=2}^{n=\infty} na_n z^n - \sum_{n=2}^{n=\infty} c_n z^n \right| - \left| 2(1-\gamma)z + \sum_{n=2}^{\infty} na_n z^n + (1-2\gamma)\sum_{n=2}^{n=\infty} c_n z^n \right| \\
&\leq \sum_{n=2}^{n=\infty} n|a_n||z|^n + \sum_{n=2}^{n=\infty} |c_n||z|^n - \left(2\,(1-\gamma)\,|z| - \sum_{n=2}^{n=\infty} n|a_n||z|^n - |1-2\gamma| \sum_{n=2}^{n=\infty} |c_n||z|^n \right) \\
&= -2(1-\gamma)|z| + \sum_{n=2}^{n=\infty} 2n|a_n||z|^n + (1+|1-2\gamma|)\sum_{n=2}^{n=\infty} |c_n||z|^n \\
&\leq \left(-2(1-\gamma) + \sum_{n=2}^{n=\infty} 2n|a_n| + (1+|1-2\gamma|)\sum_{n=2}^{n=\infty} |c_n| \right)|z| \leq 0.
\end{aligned}$$

From the above calculation we obtain that $A < 0$. Thus, we have

$$\left| zf'(z) - \frac{g(z)g(tz)}{tz} \right| < \left| zf'(z) + (1-2\gamma)\frac{g(z)g(tz)}{tz} \right| \qquad z \in \mathbb{U}$$

which is equivalent to inequality (3). Thus $f \in \mathcal{X}_t(\gamma)$ and it complete the proof of the Theorem 3.

Combining Theorem 2 and Theorem 3, we get

Theorem 4. *Let an analytic function* $f(z)$ *be given by* (1) *and* $g \in \mathcal{S}^*(1/2)$ *given by* (4)*. The inequality* (11) *holds true iff* $f \in \mathcal{X}_t(\gamma)$*.*

Remark 3. *Setting* $t = -1$ *in* (6)*, we find that*

$$\begin{aligned}
&c_{2n} = 0,\ n \in \mathbb{N}, \\
&c_3 = 2b_3 - b_2^2,\ \ c_5 = 2b_5 - 2b_2b_4 + b_3^2,\ \ c_7 = 2b_7 - 2b_2b_6 + 2b_3b_5 - b_4^2, \cdots
\end{aligned}$$

thus

$$c_{2n-1} = B_{2n-1},\ n = 2, 3, \cdots,$$

where

$$B_{2n-1} = 2b_{2n-1} - 2b_2b_{2n-2} + \cdots + (-1)^n 2b_{n-1}b_{n+1} + (-1)^{n+1}b_n^2,\ n = 2, 3, \cdots.$$

Therefore, setting $t = -1$ *in Theorem 4, we get an improved form of a known result by Kowalczyk and Les-Bomba [5].*

Theorem 5. *Let an analytic function $f(z)$ be given by* (1) *and $g \in \mathcal{S}^*(1/2)$ given by* (4)*. If $f \in \mathcal{X}_t(\gamma)$, then*

$$n^2|a_n|^2 - 4|1-\gamma|^2 \leq \left(|2\gamma-1|^2 - 1\right)\sum_{k=2}^{k=n}|c_k|^2, \tag{12}$$

where c_n is defined by (6)*.*

Proof. Since $f \in \mathcal{X}_t(\gamma)$ for some $g \in S^*(1/2)$, then the inequality (3) holds. From a simple calculation, we see that the inequality (3) is equivalent to

$$\frac{tz^2 f'(z)}{g(z)g(tz)} = \frac{1+(2\gamma-1)z\phi(z)}{1+z\phi(z)}, \quad z \in \mathbb{U},$$

where ϕ is an analytic function in $\mathbb{U}$ and $|\phi(z)| \leq 1$, for $z \in \mathbb{U}$. Then

$$\left(zf'(z) - (2\gamma-1)\frac{g(z)g(tz)}{tz}\right)z\phi(z) = \frac{g(z)g(tz)}{tz} - zf'(z).$$

Now if we put $z\phi(z) = \sum_{n=1}^{n=\infty} v_n z^n$, we see that $|z\phi(z)| \leq |z|$, for $z \in \mathbb{U}$. Thus

$$\left((2-2\gamma)z + \sum_{n=2}^{n=\infty} na_n z^n - (2\gamma-1)\sum_{n=2}^{n=\infty} c_n z^n\right)\sum_{n=1}^{n=\infty} v_n z^n = \sum_{n=2}^{n=\infty} c_n z^n - \sum_{n=2}^{n=\infty} na_n z^n, \tag{13}$$

we compare coefficients in (13). Hence we can write for $n \geq 2$

$$\left((2-2\gamma)z + \sum_{k=2}^{k=n-1} ka_k z^k - (2\gamma-1)\sum_{k=2}^{k=n} c_k z^k\right)z\phi(z) = \sum_{k=2}^{k=n} c_k z^k - \sum_{k=2}^{k=n} ka_k z^k + \sum_{k=n+1}^{k=\infty} d_k z^k.$$

Then we square the modulus of both sides of the above inequality and then we integrate along $|z| = r < 1$. After using the fact that $|z\phi(z)| \leq |z| < 1$, we obtain

$$\sum_{k=2}^{k=n}|c_k|^2 r^{2k} + \sum_{k=2}^{k=n}|ka_k|^2 r^{2k} + \sum_{k=n+1}^{k=\infty}|d_k|^2 r^{2k} < |2-2\gamma|^2 r^2 + \sum_{k=2}^{k=n-1}|ka_k|^2 r^{2k} + |2\gamma-1|^2\sum_{k=2}^{k=n}|c_k|^2 r^{2k}.$$

Letting $r \to 1$, we have

$$\sum_{k=2}^{k=n}|c_k|^2 + \sum_{k=2}^{k=n}|ka_k|^2 \leq |2-2\gamma|^2 + \sum_{k=2}^{k=n-1}|ka_k|^2 + |2\gamma-1|^2\sum_{k=2}^{k=n}|c_k|^2.$$

Hence

$$k^2|a_k|^2 - 4(1-\gamma)^2 \leq \left(|2\gamma-1|^2 - 1|\right)\sum_{k=2}^{k=n}|c_k|^2$$

Thus we have the inequality (12), which finishes the proof.

Theorem 6. *Let $0 \leq \gamma < 1$. If the function $f \in \mathcal{X}_t(\gamma)$, then*

$$|a_n| \;\leq\; \frac{1}{n}\left\{|c_n| + 2(1-\gamma)\left(1+\sum_{k=2}^{n-1}|c_k|\right)\right\}, \quad k \in \mathbb{N}. \tag{14}$$

Proof. By setting

$$\frac{1}{1-\gamma}\left(\frac{zf'(z)}{F(z)} - \gamma\right) = h(z), \quad z \in \mathbb{U}, \tag{15}$$

or equivalently

$$zf'(z) = \left[1 + (1-\gamma)(h(z)-1)\right]F(z), \tag{16}$$

we get

$$h(z) = 1 + d_1 z + d_2 z^2 + \cdots, \quad z \in \mathbb{U}, \tag{17}$$

where $\Re(h(z)) > 0$. Now using (17) and (5) in (16), we get

$$\begin{aligned}
2a_2 &= (1-\gamma)d_1 + c_2\\
3a_3 &= (1-\gamma)(d_2 + d_1c_2) + c_3\\
4a_4 &= (1-\gamma)(d_3 + d_2c_2 + d_1c_3) + c_4\\
&\vdots\\
na_n &= (1-\gamma)(d_{n-1} + d_{n-2}c_2 + \cdots + d_1c_{n-1}) + c_n.
\end{aligned}$$

Since $\Re(h(z)) > 0$, then $|d_n| \leq 2,\ n \in \mathbb{N}$. Using this property, we get

$$\begin{aligned}
2|a_2| &\leq |c_2| + 2(1-\gamma),\\
3|a_3| &\leq |c_3| + 2(1-\gamma)\left\{1 + |c_2|\right\}
\end{aligned}$$

and

$$4|a_4| \leq |c_4| + 2(1-\gamma)\left\{1 + |c_2| + |c_3|\right\},$$

respectively. Using the principle of mathematical induction, we obtain (14). This completes proof of Theorem 6.

Corollary 7. *Let $0 \leq \gamma < 1$. If the function $f \in \mathcal{X}_t(\gamma)$, then*

$$|a_n| \;\leq\; 1 + (n-1)(1-\gamma). \tag{18}$$

Proof. From remark 1.1, we know that $F(z) \in \mathcal{S}^*$, thus $|c_n| \leq n$. The assertion (18), can now easily derived from Theorem 6.

Remark 4. *Setting $t = -1$ in Corollary 7, we get the corresponding result due to Geo and Zhou [1].*

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