Acta Universitatis Apulensis No. 38/2014
ISSN: 1582-5329 pp. 45-53

UNIFICATION OF π-GENERALIZED CLOSED SETS BY HEREDITARY CLASSES IN GENERALIZED TOPOLOGICAL SPACES

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Abstract. In this paper, we introduce the notions of $\pi g\mu^\star$-closed and $\pi g\mu^\star$-open sets by using the notion of π-open sets. We generalize many concepts which is defined in ideal topological spaces and topological spaces by using these new notions. Also we study quasi $\mu_{\mathcal{H}}$- normality and characterizations of quasi $\mu_{\mathcal{H}}$- normal spaces are obtained. Several preservation theorems for quasi $\mu_{\mathcal{H}}$- normal spaces are given.

2000 *Mathematics Subject Classification*: 54A05, 54C05

Keywords: generalized topology, hereditary class, $\pi g\mu^\star$-closed, quasi μ_g-$\mathcal{H}$-normal space.

1. Introduction and Preliminaries

The idea of generalized topology and hereditary classes was introduced and studied by Császár [5, 6]. He generalized ideal topology on a set by using these structures. In this paper, we introduce the notions of $\pi g\mu^\star$-closed and $\pi g\mu^\star$-open sets by using the notion of π-open sets. We generalize many concepts which is defined in ideal topological spaces and topological spaces by using these new notions.

Let A be a subset of a topological space X. The closure of A and the interior of A are denoted by Cl(A) and Int(A), respectively. A subset A of a topological space (X, τ) is said to be regular open [24](resp. regular closed) if A = Int(Cl(A)) (resp. A = Cl(Int(A))). The finite union of regular open sets is said to be π-open [28] in (X, τ). The complement of a π-open set is π-closed. A subset A of a topological space (X, τ) said to be semi-open [12] (resp., α-open [16], pre-open [14], b-open [2], β-open [1]) if A $\subset$ Cl(Int(A)) (resp. A $\subset$ Int(Cl(Int(A))), A $\subset$ Int(Cl(A)), A $\subset$ Int(Cl(A))$\cup$ Cl(Int(A)), A $\subset$ Cl(Int(Cl(A)))). The family of all semi-open (resp. α-open, pre-open, b-open, β-open) sets in (X, τ) is denoted by SO(X) (resp. $\alpha O(X)$, PO(X), BO(X), βO(X)). A function f:$(X, \tau) \to (Y, \sigma)$ is said to be m-π-closed [9] if f(V) is π-closed in (Y, σ) for every π-closed in (X, τ). A function f:$(X, \tau) \rightarrow (Y, \sigma)$ is said π-continuous [8] if $f^{-1}(V)$ is π-closed in (X, τ) for every closed set in (Y, σ).

An ideal topological space is a topological space (X, τ) with an ideal I on X, and is denoted by (X, τ, I). $A^*(I) = \{x \in X \mid U \cap A \notin I \mathit{for\ each\ open\ neighborhood\ U\ of\ x}\}$ is called the local function of A with respect to I and τ [11]. When there is no chance for confusion $A^*(I)$ is denoted by A^*. For every ideal topological space (X, τ, I), there exists a topology τ^*(I), finer than τ. Observe additionally that $Cl^*(A) = A^* \cup A$ defines a Kuratowski closure operator for $\tau^*(I)$ [27]. A subset A of an ideal topological space (X, τ, I) is said to be *semi**-I-open [10] if A $\subset$ $Cl(Int^\star(A))$. The family of all *semi**-I-open sets in (X, τ, I) is denoted by $S^\star IO(X)$.

Let X be a non-empty set and exp X denote the power set of X. We call a class μ $\subset$ exp X a generalized topology [5] (briefly, GT) if $\emptyset \in \mu$ and the union of elements of μ belongs to μ. And let us say that a hereditary class $\mathcal{H}$ [6] on X is a class $\emptyset \neq \mathcal{H} \subset$ X satisfying A $\subset$ B, B $\in \mathcal{H}$ implies A $\in \mathcal{H}$. If μ is a GT on X and A $\subset$ X, x $\in$ X then x $\in A^\star_\mu$ [6] iff x $\in$ M $\in \mu \Rightarrow$ M $\cap$ A $\notin \mathcal{H}$. There is a GT $\mu^\star$ [6] such that $c^\star_\mu(A)$ = A $\cup$ $A^\star_\mu$ is intersection of all $\mu^\star$-closed supersets of A; M $\in \mu^\star$ iff X - M = $c^\star_\mu$(X - M).

If one takes $\mathcal{H} = \emptyset$, then $c_{\mu^\star}$ becomes c_μ. If one takes τ as GT and $\mathcal{H} = \emptyset$, then $c_{\mu^\star}$ becomes the usual closure operator. Similarly $c_{\mu^\star}$ becomes scl (resp. pcl, bcl, βcl) if $\mu^\star$ stands for SO(X) (resp. PO(X), BO(X), βO(X)). Likewise, if one takes τ as GT and $\mathcal{H}$ = I, then $c_{\mu^\star}$ becomes closure operator for $\tau^\star$(I). Likewise, $c_{\mu^\star}$ becomes s*cl if μ stands for S*IO(X).

Given a topological space (X, τ) and a GT μ on X, (X, τ) is said to be μ_g-normal [18] if for any two disjoint closed sets A and B there exist two disjoint μ-open sets U and V such that A $\subset$ U and B $\subset$ V.

2. πG$\mu^\star$-closed sets

Definition 1. *Let μ be a GT and $\mathcal{H} \neq \emptyset$ be a hereditary class on a topological space (X, τ). A subset A of X is called a π generalized $\mu^\star$-closed set (or simply $\pi g\mu^\star$-closed)if $c_{\mu^\star}(A) \subset U$ whenever $A \subset U$ and U is π-open.*

The complement of a πg$\mu^\star$-closed set is said to be πg$\mu^\star$-open.

Remark 1. (*a*) *Let μ be a GT and $\mathcal{H} \neq \emptyset$ be a hereditary class on topological space (X, τ). Then every $\pi g\mu^\star$-closed set reduces to πg-closed [8] (resp., πgs-closed [3], πgp-closed [19], πgb-closed [22], $\pi g\beta$-closed [25]) if μ is taken to be τ (resp., SO(X), PO(X), BO(X), βO(X)) and $\mathcal{H} = \emptyset$.*
(*b*) *Let μ be a GT and $\mathcal{H} \neq \emptyset$ be a hereditary class on X. Then every $\pi g\mu^\star$-closed*

set reduces to $I_{\pi g}$-closed [20] ($I_{\pi gs^\star}$-closed [10]) if $\mu^\star$ is taken to be $\tau^\star$ ($S^\star IO(X)$) and $\mathcal{H}$= I.

Theorem 1. Every g_μ-closed set is πg$\mu^\star$-closed.

Proof. It is obvious that every π-open set is open.

Remark 2. *The following example shows that the reverse of Theorem 2.1 is not true.*

Example 1. *Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{a\}, \{c, d\}, \{a, c, d\}\}$, $\mathcal{H} = \{\emptyset, \{c\}\}$ and $\mu = \{X, \emptyset, \{a, c\}, \{b, d\}\}$. Then the set $\{a, d\}$ is $\pi g\mu^\star$-closed but not g_μ-closed.*

Remark 3. *Finite intersection (union) of $\pi g\mu^\star$-closed sets need not to be $\pi g\mu^\star$-closed by the following examples.*

Example 2. *Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{a, c\}, \{b, d\}\}$, $\mathcal{H} = \{\emptyset, \{a\}\}$ and $\mu = \{X, \emptyset, \{a, b\}, \{a, c\}\}$. $A = \{b, c\}$ and $B = \{c, d\}$. Clearly A and B are $\pi g\mu^\star$-closed sets but $A \cap B$ is not $\pi g\mu^\star$-closed.*

Example 3. *Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$, $\mathcal{H} = \{\emptyset, \{a\}\}$ and $\mu = \{X, \emptyset, \{a, d\}\}$. $A = \{a, c\}$ and $B = \{b\}$. Clearly A and B are $\pi g\mu^\star$-closed sets but $A \cup B$ is not $\pi g\mu^\star$-closed.*

Theorem 1. *Let μ be a GT and $\mathcal{H} \neq \emptyset$ be a hereditary class on a topological space (X, τ). If A is $\pi g\mu^\star$-closed, B is π-closed and $\mu^\star$-closed then $A \cap B$ is $\pi g\mu^\star$-closed.*

Proof. Let U be π-open such that A $\cap$ B $\subset$ U. Then A $\subset$ U $\cup$(X $\setminus$ B). Since A is πg$\mu^\star$-closed and B is π-closed then $c_{\mu^\star}(A) \subset$ (U $\cup$(X $\setminus$ B)). Hence $c_{\mu^\star}$(A $\cap$ B) $\subset$ (U $\cup$(X $\setminus$ B)) . Since B is $\mu^\star$-closed, $c_{\mu^\star}$(A $\cap$ B) $\subset$ U. Hence A $\cap$ B is πg$\mu^\star$-closed.

Theorem 2. *Let μ be a GT and $\mathcal{H} \neq \emptyset$ be a hereditary class on a topological space (X, τ). For every $A \in \mathcal{H}$, A is $\pi g\mu^\star$-closed.*

Proof. Let A $\subset$ U where U is π-open. Since $A^\star_\mu = \emptyset$ for every A $\in \mathcal{H}$, $c_{\mu^\star}(A)$ = A $\cup$ $A^\star_\mu$ = A $\subset$ U. Therefore A is πg$\mu^\star$-closed.

Theorem 3. *Let μ be a GT and $\mathcal{H} \neq \emptyset$ be a hereditary class on a topological space (X, τ). For every subset A of X, $A^\star_\mu$ is $\pi g\mu^\star$-closed.*

Proof. Let $A^\star_\mu \subset$ U where U is π-open. Since $(A^\star_\mu)^\star_\mu \subseteq A^\star_\mu$, we have $c_{\mu^\star}(A^\star_\mu) \subset$ U . Hence $A^\star_\mu$ is πg$\mu^\star$-closed.

Theorem 4. *Let μ be a GT and $\mathcal{H} \neq \emptyset$ be a hereditary class on a topological space (X, τ). If A is $\pi g\mu^\star$-closed, then $c_{\mu^\star}(A) \setminus A$ does not contain any nonempty π-closed set.*

Proof. Let F be π-closed subset of X, such that F $\subset c_{\mu^\star}(A) \setminus$ A where A is πg$\mu^\star$-closed. Then $c_{\mu^\star}(A) \subset$ (X $\setminus$ F). Thus F $\subset$ (X $\setminus$ $c_{\mu^\star}(A)$) $\cap$ $c_{\mu^\star}(A)$ and hence F = $\emptyset$.

Theorem 5. *Let μ be a GT and $\mathcal{H} \neq \emptyset$ be a hereditary class on a topological space (X, τ) and $A \subset B \subset c_{\mu^\star}(A)$, where A is $\pi g\mu^\star$-closed. Then B is $\pi g\mu^\star$-closed.*

Proof. Let B $\subset$ U and U is π-open. Since A is πg$\mu^\star$-closed and B $\subset$ U, then $c_{\mu^\star}(A)$ $\subset$ U. Now, A $\subset$ B $\subset$ $c_{\mu^\star}(A)$, $c_{\mu^\star}(A) = c_{\mu^\star}(B)$ and hence $c_{\mu^\star}(B) \subset$ U. Thus B is πg$\mu^\star$-closed.

Theorem 6. *Let μ be a GT and $\mathcal{H} \neq \emptyset$ be a hereditary class on a topological space (X, τ). Every π-open set is $\mu^\star$-closed set if and only if every subset of X is $\pi g\mu^\star$-closed.*

Proof. Suppose every π-open set is μ-$\star$-closed. Let A be a subset of X. If U is π-open such that A $\subset$ U, then $A^\star_\mu \subset U^\star_\mu \subset$ U and $c_{\mu^\star}(A) \subset$ U. So A is πg$\mu^\star$-closed.

Conversely, suppose that every subset of X is πg$\mu^\star$-closed. If U is π-open then by hypothesis, U is πg$\mu^\star$-closed and so $c_{\mu^\star}(U) \subset$ U. Thus, $U^\star_\mu \subset$ U and so U is $\mu^\star$-closed.

Theorem 7. *Let μ be a GT and $\mathcal{H} \neq \emptyset$ be a hereditary class on a topological space (X, τ). For each $x \in X$, either $\{x\}$ is π-closed or $\{x\}^c$ is $\pi g\mu^\star$-closed.*

Proof. Suppose that $\{x\}$ is not π-closed, then $\{x\}^c$ is not π-open and only π-open set containing $\{x\}^{\ c}$ is set X itself. So $\{x\}^c$ is πg$\mu^\star$-closed.

Theorem 8. *Let μ be a GT and $\mathcal{H} \neq \emptyset$ be a hereditary class on a topological space (X, τ). If A is $\pi g\mu^\star$-closed in X, such that $A \subset Y \subset X$, then A is $\pi g\mu^\star$-closed in Y.*

Proof. Let U be a π-open set in Y such that A $\subset$ U, then A $\subset$ U = V$\cap$ Y where V is π-open in X. Since A is πg$\mu^\star$-closed in X, $c_{\mu^\star}(A) \subset$ V . Therefore $c_{\mu^\star_Y}(A) \subset$ U. Then A is πg$\mu^\star$-closed in Y.

Theorem 9. *Let μ be a GT and $\mathcal{H} \neq \emptyset$ be a hereditary class on a topological space (X, τ). A subset A of X is $\pi g\mu^\star$-open if and only if $F \subset int_{\mu^\star}(A)$ whenever $F \subset A$ and F is π-closed.*

Proof. Let F $\subset$ A and F be π-closed. Then (X $\setminus$ A) $\subset$ (X $\setminus$ F) and X $\setminus$ F π-open. Since X $\setminus$ A is πg$\mu^\star$-closed ($c_{\mu^\star}$(X $\setminus$ A)) $\subset$ (X $\setminus$ F) . So F $\subset int_{\mu^\star}(A)$. Conversely suppose that F $\subset int_{\mu^\star}(A)$ whenever F $\subset$ A and F is π-closed. Let X$\setminus$A $\subset$ U where U is π-open. Then X $\setminus$U $\subset$ A. Then by hypothesis X $\setminus$U $\subset int_{\mu^\star}(A)$ and hence $c_{\mu^\star}(X \setminus A) \subset$ U. Therefore A is πg$\mu^\star$-open.

Theorem 10. *Let μ be a GT and $\mathcal{H} \neq \emptyset$ be a hereditary class on a topological space (X, τ). If a subset of X is $\pi g\mu^\star$-open then $U = X$ whenever U is π-open and $int_{\mu^\star}(A) \cup (X \setminus A) \subset U$.*

Proof. Let U be π-open and $int_{\mu^\star}(A) \cup$ (X $\setminus$ A)$\subset$ U for πg$\mu^\star$-open A. Then X $\setminus$ U $\subset$ (X $\setminus$ $int_{\mu^\star}(A)$) $\cap$ A. Since X $\setminus$ A is πg$\mu^\star$-closed and by the Theorem 2.5 X $\setminus$ U $= \emptyset$, hence X = U.

3. Quasi μ_g-$\mathcal{H}$-Normal Spaces

Definition 2. *Let μ be a GT and $\mathcal{H} \neq \emptyset$ be a hereditary class on a topological space (X, τ). A topological space is called a quasi μ_g-$\mathcal{H}$-normal space if for every pair of disjoint π-closed sets A and B of X, there exist disjoint μ-open sets U and V such that $A \setminus U \in \mathcal{H}$ and $B \setminus V \in \mathcal{H}$.*

Remark 4. *Let μ be a GT and $\mathcal{H} = \{\emptyset\}$ on a topological space (X, τ). Then every quasi μ_g-$\mathcal{H}$-normal space reduces to be quasi-normal [8](resp., quasi-s-normal [4], quasi-p-normal [26]) space if μ is taken to be τ (resp., SO(X), PO(X)).*

Theorem 11. *Every μ_g-normal space is a quasi μ_g-$\mathcal{H}$-normal space.*

Proof. It is obvious by every μ-open set is $\mu^\star$-open.

The following example shows that the reverse of Theorem 3.1 is not true.

Example 4. *Observe that the Countable Extension Topological space [Example 63, [23]] in which X is real line, and if τ_1 is the Euclidean topology on X and τ_2 is the topology of countable complements on X, we define τ to be the smallest topology generated by $\tau_1 \cup \tau_2$. Let $\mu = \{\emptyset\} \cup \{[n, \infty) | n \in N\} \cup \{(-\infty, n] | n \in N\}$ be generalized topology on (X, τ) and $\mathcal{H} = \{H | H \subseteq [a, b]\}$ be hereditary on X. Then X is a quasi μ_g-$\mathcal{H}$-normal space but not μ_g-normal space.*

Theorem 12. *Let μ be a GT and $\mathcal{H} \neq \emptyset$ be a hereditary class on a topological space (X, τ). Then the followings are equivalent:*
(*a*) *X is a quasi μ_g-$\mathcal{H}$-normal space.*
(*b*) *For every π-closed set F and π-open set G containing F, there exists a μ-open set V such that $F \setminus V \in \mathcal{H}$ and $c_{\mu^\star}(V) \setminus G \in \mathcal{H}$.*
(*c*) *For each pair of disjoint π-closed sets A and B, there exists an μ-open set U such that $A \setminus U \in \mathcal{H}$ and $c_{\mu^\star}(U) \cap B \in \mathcal{H}$.*

Proof. (a) $\Rightarrow$ (b) Let F be a π-closed and G be a π-open subset of X. Since X $\setminus$G is π-closed and F$\subset$G, F$\cap$ (X$\setminus$G)=$\emptyset$. X is a quasi μ_g-$\mathcal{H}$-normal space, so there

exist disjoint μ-open sets U and V such that F\V$\in \mathcal{H}$ and (X\G)\U$\in \mathcal{H}$. Then $c_{\mu^\star}(V) \subset$X\U and (X\G)$\cap c_{\mu^\star}(V) \subset$(X\G)$\cap$(X\U). Hence $c_{\mu^\star}(V)$\G$\in \mathcal{H}$.
(b) $\Rightarrow$ (c) Obvious by the hypothesis.
(c) $\Rightarrow$ (a) Let A and B be disjoint π-closed sets. By the hypothesis there exists a μ-open set U such that A\U$\in \mathcal{H}$ and $c_{\mu^\star}(U)\cap$B $\in \mathcal{H}$. Let V= X\$c_{\mu^\star}(U)$. Since V is μ-open and U$\cap$V=$\emptyset$, X is a quasi μ_g-$\mathcal{H}$-normal space.

Theorem 13. *Let μ be a GT and $\mathcal{H}\neq\emptyset$ be a hereditary class on (X,τ) topological space. If X is a quasi μ_g-$\mathcal{H}$- normal space then for every pair of disjoint π-closed sets A and B of X, there exist disjoint $\pi g\mu^\star$-open sets U and V such that $A\setminus U \in \mathcal{H}$ and $B\setminus V \in \mathcal{H}$.*

Proof. It is obvious by every μ-open set is πg$\mu^\star$-open.

Theorem 14. *Let μ be a GT and $\mathcal{H}$=$\{\emptyset\}$ be a hereditary class on a (X,τ) topological space. Then the following are equivalent.*
(*a*) *X is a quasi μ_g-$\mathcal{H}$-normal space*
(*b*) *If for every pair of disjoint π-closed sets A and B of X, there exist disjoint $\pi g\mu^\star$-open sets U and V such that $A \setminus U \in \mathcal{H}$ and $B \setminus V \in \mathcal{H}$.*

Proof. (a) $\Rightarrow$ (b) It is obvious by the previous theorem.
(b) $\Rightarrow$ (a) Let A and B be disjoint π-closed. By the hypothesis there exist U and V are disjoint πg$\mu^\star$-open subsets of X such that A \U=$\emptyset$ B \V=$\emptyset$. Then A $\subset$ U. Since U is πg$\mu^\star$-open, A $\subset int_{\mu^\star}(U)$ by Theorem 2.10. Similarly B $\subset int_{\mu^\star}(V)$. Finally, since $int_{\mu^\star}(U)$ and $int_{\mu^\star}(V)$ are μ-open sets and $\mathcal{H}$= $\{\emptyset\}$, X is a quasi μ_g-$\mathcal{H}$- normal space.

Definition 3 ([13]). *Let (X,μ) and (Y,λ) be GTSs, then a function $f : X \to Y$ is called (μ,λ)-open if f(G) $\in \lambda$ for each $G \in \mu$.*

Lemma 1. If $\mathcal{H}\neq\emptyset$ is a hereditary class on X and f : X $\to$ Y is a function, then f($\mathcal{H}$) = $\{f(H)|H \in \mathcal{H}\}$ is a hereditary class on Y.

Theorem 15. *Let μ be a GT and $\mathcal{H}\neq\emptyset$ be a hereditary class on a (X,τ),λ be a GT on (Y,σ) and f:X$\to$Y is a bijection, π-continuous and (μ,λ)-open. If X is a quasi μ_g-$\mathcal{H}$- normal space, then Y is a quasi λ_g-f($\mathcal{H}$)-normal space.*

Proof. Let A and B be disjoint π-closed subsets of Y, since f is π-continuous function $f^{-1}(A)$, $f^{-1}(B)$ are π-closed subsets of X. Since X is quasi μ_g-$\mathcal{H}$- normal space, there exist disjoint μ-open sets U and V in X such that $f^{-1}(A) \setminus U \in \mathcal{H}$ and $f^{-1}(B)$\V $\in \mathcal{H}$. Then f(f^{-1}(A)\ U)$\in$ $f(\mathcal{H})$ and f(f^{-1} (A)) \ f(U)$\in$ $f(\mathcal{H})$. Because of the hereditary of $\mathcal{H}$, A \ f(U)$\in$ f($\mathcal{H}$). Similarly, B \ f(U)$\in$f($\mathcal{H}$). Since f(U) and f(V) are disjoint λ-open subsets of Y, it follows that Y is a quasi λ_g-f($\mathcal{H}$)- normal space.

Definition 4 ([5]). *Let (X,μ) and (Y,λ) be GTSs, then a function $f : X \to Y$ is called (μ,λ)-continuous if $f^{-1}(G) \in \mu$ for each $G \in \lambda$.*

Lemma 2. If $\mathcal{H} \neq \emptyset$ is a hereditary class on Y and f : X $\to$ Y is a function, then f $^{-1}(\mathcal{H}) = \{f^{-1}(H)|H \in \mathcal{H}\}$ is a hereditary class on X.

Theorem 16. *Let μ be a GT and $\mathcal{H} \neq \emptyset$ be a hereditary class on (X,τ),λ be a GT on (Y,σ) and f:X $\to$ Y is an injection, m-π-closed and (μ,λ)-continuous. If Y is a quasi λ_g-$\mathcal{H}$-normal space, then X is a quasi μ_g-$f^{-1}(\mathcal{H})$-normal space.*

Proof. Let A and B be disjoint π-closed subsets of X, since f is m-π-closed injection, f(A) and f(B) are disjoint π-closed subset of Y. Since Y is quasi $\lambda_{\mathcal{H}}$- normal space, there exist disjoint λ-open U and V such in Y that f(A) \U $\in \mathcal{H}$ and f(B) \V $\in \mathcal{H}$. Then f^{-1}(f(A)\U) $\in f^{-1}(\mathcal{H})$ and f^{-1}(f(A))\f^{-1}(U) $\in$ f$^{-1}(\mathcal{H})$).Then A \f^{-1}(U)$\in$ f$^{-1}(\mathcal{H})$). Similarly, B\f^{-1}(V)$\in$ f$^{-1}(\mathcal{H})$). Since f is (μ,λ)-continuous, f^{-1}(U) and f^{-1}(V) are disjoint μ-open subsets of X. It follows that X is a quasi μ_g-$f^{-1}(\mathcal{H})$-normal space.

Lemma 3. If $\mathcal{H} \neq \emptyset$ is a hereditary class on X and Y is a subset of X, then $\mathcal{H}_Y = \{Y \cap H|H \in \mathcal{H}\} = \{H \in \mathcal{H}|H \subset Y\}$ is a hereditary class on Y.

Theorem 17. *Let μ be a GT and $\mathcal{H} \neq \emptyset$ be a hereditary class on a (X,τ) topological space. If X is a quasi μ_g-$\mathcal{H}$- normal space and Y$\subset$X is π-closed, then Y is a quasi μ_g-$\mathcal{H}_Y$- normal space.*

Proof. Let A and B be disjoint π-closed subsets of Y. Since Y is π-closed A and B are disjoint π-closed subsets of X. By hypothesis, there exist disjoint open sets U and V such that A\U$\in \mathcal{H}$ and B\V $\in \mathcal{H}$. If A\U = H$\in \mathcal{H}$ and B\V=$G \in \mathcal{H}$, then A$\subset$U$\cup H$ and B$\subset$V$\cup G$. Since A$\subset$Y, A$\subset$Y$\cap(U \cup H)$ and so A$\subset$(Y$\cap$ H)$\cup$(Y$\cap$U). Therefore A\(Y$\cap$U)$\subset$(Y$\cap H$)$\in \mathcal{H}_Y$. Similarly, B\(Y$\cap$V)$\subset$(Y$\cap$ G)$\in \mathcal{H}_Y$. If U_1=Y$\cap$U and V_1=Y$\cap$V, then U_1 and V_1 are disjoint μ_Y-open sets such that A\$U_1 \in \mathcal{H}_Y$ and B\$V_1 \in \mathcal{H}_Y$. Hence Y is a quasi μ_g-$\mathcal{H}_Y$- normal space.

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