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SKEW ELEMENTS IN N-SEMIGROUPS

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Abstract. We introduce the notions of a polyadic inverse and a skew element in n-groupoids and prove that an n-semigroup has skew elements iff it is H-derived from a monoid with invertible elements (a generalization of classical Gluskin-Hosszú theorem). Based on this result some properties of skew elements are presented.

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1. Introduction

A nonempty set A together with one n-ary operation $\alpha : A^n \to A$, $n > 2$, is called an *n-groupoid* and is denoted by (A, α).

Traditionally in the theory of n-groupoids we use the following abbreviated notation: the sequence $x_i, \ldots, x_j$ is denoted by x_i^j (for $j < i$ this symbol is empty). If $x_{i+1} = x_{i+2} = \cdots = x_{i+k} = x$ then instead of x_{i+1}^{i+k} we write $(x)^k$. For $k \leq 0$ $(x)^k$ is the empty symbol.

Let (A, α) be an n-groupoid. We say that this groupoid is *i-solvable* if for all $a_1, \ldots, a_n$, $b \in A$ there exists $x_i \in A$ such that

$$\alpha(a_1^{i-1}, x_i, a_{i+1}^n) = b \qquad (1)$$

If this solution is unique, we say that this n-groupoid is *uniquely i-solvable*. An n-groupoid which is uniquely i-solvable for every $i = 1, 2, \ldots, n$ is called an *n-quasigroup*.

We say that the operation α is (i, j)-*associative* if

$$\alpha(a_1^{i-1}, \alpha(a_i^{i+n-1}), a_{i+n}^{2n-1}) = \alpha(a_1^{j-1}, \alpha(a_j^{j+n-1}), a_{j+n}^{2n-1}) \qquad (2)$$

holds for all $a_1, \ldots, a_{2n-1} \in A$. If α is (i, j)-associative for every $i, j \in \{1, \ldots, n\}$ then it is called *associative*.

An n-groupoid with an associative operation is called an n-semigroup.

An n-semigroup which is also an n-quasigroup is called an *n-group* (cf. [10]).

Let (A,α) be an n-groupoid and let $M \subseteq A$ be a nonempty set. We say that α is *M-i-cancellative* if

$$\alpha(m_1^{i-1}, x, m_{i+1}^n) = \alpha(m_1^{i-1}, y, m_{i+1}^n) \Rightarrow x = y \tag{3}$$

for all $m_1, \ldots, m_n \in M$. If α is M-i-cancellative for every $i \in \{1, \ldots, n\}$ then it is called *M-cancellative.*

In the theory of n-groupoids the following identities

$$\alpha(a_1, a_2^{n-1}, a_n) = \alpha(a_n, a_2^{n-1}, a_1) \tag{4}$$

and

$$\alpha(\alpha(a_{11}^{1n}), \ldots, \alpha(a_{n1}^{nn})) = \alpha(\alpha(a_{11}^{n1}), \ldots, \alpha(a_{1n}^{nn})) \tag{5}$$

play a very important role.

The first of them is called *semicommutativity* or $(1,n)$-*commutativity.* The second is a special case of the abelian law for general algebras (see [6]).

Glazek and Gleichgewicht [4] proved

Theorem 1. *Each semicommutative n-semigroup is abelian.*

and

Theorem 2. *An n-group is abelian if and only if it is semicommutative.*

2. Polyadic inverses

Let (A,α) be an n-groupoid. The unit in a groupoid has several generalizations. One of them is the following (see [7]): an $(n-1)$-tuple a_1^{n-1} of elements from A is called a *left* (*right*) *identity* if

$$\alpha(a_1^{n-1}, x) = x \quad (\alpha(x, a_1^{n-1}) = x) \tag{6}$$

for all $x \in A$. A *lateral identity* is one which is both a left and right identity. It is called an *identity* if any cyclic permutation of it is a lateral identity.

Now we extend the notion of the inverse element in a groupoid.

Definition 1. *An element $a \in A$ is called a* ***polyadic inverse*** *of the ordered system $(a_1, \ldots, a_{n-2})$ if*

$$\alpha(a, a_1^{n-2}, x) = \alpha(x, a_1^{n-2}, a) = x \tag{7}$$

for all $x \in A$.

An ordered system has at most one polyadic inverse. Let a and $\tilde{a}$ be a polyadic inverses of $(a_1, \ldots, a_{n-2})$. Then $\alpha(\tilde{a}, a_1^{n-2}, a) = \tilde{a}$ since a is a polyadic inverse of $(a_1, \ldots, a_{n-2})$. But $\alpha(\tilde{a}, a_1^{n-2}, a) = a$ since $\tilde{a}$ is a polyadic inverse too. Therefore the notation

$$a = (a_1, \ldots, a_{n-2})^{-1}$$

is consistent. If $(a_1, \ldots a_{n-2})$ has polyadic inverse we say that it is **polyadic invertible**.

Let (A, α) be an n-semigroup *derived* from a monoid $(A, \cdot)$, i.e.

$$\alpha(x_1^n) = x_1 \cdot x_2 \cdot \ldots \cdot x_n.$$

Then $(a_1, \ldots a_{n-2})$ is polyadic invertible in (A, α) iff the product $a_1 \cdot a_2 \cdot \ldots \cdot a_{n-2}$ is invertible in $(A, \cdot)$ and

$$(a_1, a_2, \ldots, a_{n-2})^{-1} = (a_1 a_2 \ldots a_{n-2})^{-1}.$$

In particular, for $n = 3$ an element $a \in A$ is polyadic invertible in (A, α) iff is invertible in $(A, \cdot)$.

In [8] we proved

Theorem 3.*Let* (A, α) *be an* n*-semigroup and* $a = (a_1, \ldots, a_{n-2})^{-1}$. *Then both* $(a, a_1, \ldots, a_{n-2})$ *and* $(a_1, \ldots, a_{n-2}, a)$ *are lateral identities.*

A result of Monk and Sioson [7] can be reformulated as

Theorem 4.*An* n*-semigroup is an* n*-group iff every* $(n-2)$*-tuple is polyadic invertible.*

Definition 2.*The polyadic inverse* $(a, \ldots, a)^{-1}$ *is called the* ***skew element to*** a *and is denoted by* $\bar{a}$. *In this case* a *is called* ***skewable****.*

Let (A, α) be an n-semigroup derived from a monoid $(A, \cdot)$. The element a is skewable iff a^{n-2} is invertible in $(A, \cdot)$. Then $\bar{a} = a^{2-n}$. It is easy to see that a is skewable iff a is invertible in $(A, \cdot)$.

In [8] we proved

Theorem 5.*If* $\bar{a}$ *is the skew element to* a *in the* n*-semigroup* (A, α) *then* $(a)^{n-2}, \bar{a}$ *is an identity.*

Corollary 1.*Let a be a skewable element in the n-semigroup (A,α) with $n>3$. Then*

$$((a)^i,\bar{a},(a)^{n-3-i})^{-1}=a \qquad (8)$$

for all i, $0\leq i\leq n-3$.

A result of Dudek, Glazek and Gleichgewicht [1] can be reformulated as

Theorem 6.*An n-semigroup is an n-group iff all its elements are skewable.*

3. H-derived n-semigroups

We extend a definition of [2] (see also [8], [9]).

An n-groupoid (A,α) is said to be *H-derived from a monoid* $(A,\cdot)$ if there exist an invertible element $a\in A$ and an automorphism f of $(A,\cdot)$ such that

$$f(a)=a, \qquad (9)$$

$$f^{n-1}(x)=a\cdot x\cdot a^{-1} \qquad (10)$$

and

$$\alpha(x_1^n)=x_1\cdot f(x_2)\cdot\ldots\cdot f^{n-1}(x_n)\cdot a \qquad (11)$$

We also say that (A,α) is $H_{<f,a>}$ *derived from* $(A,\cdot)$ and it is denoted by $(A,\alpha)=H_{<f,a>}(A_1,\cdot)$.

If $(A,\cdot)$ is a group we have the notion of H-derived from a group (see [2]). The Gluskin-Hosszú theorem say (see [2], [5]): an n-groupoid is an n-group iff it is H-derived from a group.

We will describe the H-derived n-semigroups (see [9]).

Theorem 7. *Let (A,α) be an n-groupoid H-derived from a monoid,*

$$(A,\alpha)=H_{<f,a>}(A,\cdot).$$

Then (A,α) is an n-semigroup with skewable elements.

Proof. (For details see [9]) The equalities (9) and (10) imply the associativity of the operation α. If e is the unit of $(A,\cdot)$ then $a^{-1}=\bar{e}$.

Let (A,α) be an n-groupoid and $a_1,\ldots,a_{n-2}$ be fixed elements of A. Then $(A,\cdot)$ where $x\cdot y=\alpha(x,a_1^{n-2},y)$ is called a *binary retract* of (A,α) and is denoted by $(A,\cdot)=\mathrm{ret}_{a_1^{n-2}}(A,\alpha)$.

Suppose that a is a skewable element in an n-semigroup (A, α). We will use some ideas from the proof of Gluskin-Hosszú theorem given by Sokolov [11]. For details see [9].

$(A, \cdot) = \mathrm{ret}_{(a)^{n-2}}(A, \alpha)$ is a monoid having $\bar{a}$ as unit. The mapping $f : A \to A$, $f(x) = \alpha(\bar{a}, x, (a)^{n-2})$ is an automorphism of $(A, \cdot)$. For $u = \alpha((\bar{a})^n)$ we have

$$\begin{aligned} &f(u) = u, \\ &f^{n-1}(x) \cdot u = u \cdot x \end{aligned} \tag{12}$$

and

$$\alpha(x_1^n) = x_1 \cdot f(x_2) \cdot \ldots \cdot f^{n-1}(x_n) \cdot u.$$

Since $f(a) = a$ from (12) we get $a \cdot u = u \cdot a$. From

$$\bar{a} = \alpha(\bar{a}, \bar{a}, (a)^{n-2}) = \bar{a} \cdot \bar{a} \cdot a^{n-2} \cdot u = a^{n-2}u = u \cdot a^{n-2}$$

it follows that u is invertible in $(A, \cdot)$. Now from (12) we get

$$f^{n-1}(x) = u \cdot x \cdot u^{-1} \tag{13}$$

In conclusion $(A, \alpha) = H_{<f,u>}(\mathrm{ret}_{(a)^{n-2}}(A, \alpha))$.

Thus the following theorem is true

Theorem 8. *Each n-semigroup with skewable elements is H-derived from a monoid.*

Now from the above theorems we obtain the following characterization of H-derived n-semigroups.

Theorem 9. *An n-semigroup is H-derived from a monoid iff it has skewable elements.*

We finish this section by the following characterization of semicommutative n-semigroups.

Theorem 10. [9]*Let $(A, \alpha) = H_{<f,a>}(A, \cdot)$. Then (A, α) is semicommutative iff $(A, \cdot)$ is commutative.*

Proof. Suppose that (A, α) is semicommutative. From $\alpha(x_1(e)^{n-3}, a^{-1}, y) = x \cdot y$ where e is the unit of $(A, \cdot)$ we obtain, taking into account (4), $x \cdot y = y \cdot x$ for all $x, y \in A$.

Let now $(A, \cdot)$ be commutative. Then

$$\begin{aligned}\alpha(x_1, x_2^{n-1}, x_n) &= x_1 \cdot f(x_2) \cdot \ldots \cdot f^{n-2}(x_{n-1}) \cdot a \cdot x_n \\ &= x_n \cdot f(x_2) \cdot \ldots \cdot f^{n-2}(x_{n-1}) \cdot a \cdot x_1 \\ &= \alpha(x_n, x_2^{n-1}, x_1),\end{aligned}$$

i.e., (A, α) is semicommutative.

4. Properties of skew elements

Let (A, α) be an n-semigroup.

Theorem 11.*If* $a = (a_1, \ldots, a_{n-2})^{-1}$ *then* a *is skewable.*

Proof. The technical details were omitted.

It is easy to prove that $(A, \cdot) = \mathrm{ret}_{a_1^{n-2}}(A, \alpha)$ is a monoid with a as unit element. The mapping $f : A \to A$, $f(x) = \alpha(a, x, a_1^{n-2})$ is an automorphism of $(A, \cdot)$ ($f^{-1}(x) = \alpha(a_1^{n-2}, x, a)$. For $u = \alpha((a)^n)$ we have $f(u) = u$,

$$f^{n-1}(x) \cdot u = u \cdot x \tag{14}$$

and

$$\alpha(x_1^n) = x_1 \cdot f(x_2) \cdot \ldots \cdot f^{n-1}(x_n) \cdot u.$$

We prove that u is invertible. From $a = \alpha(a, a_1^{n-2}, a)$ we get

$$a = f(a_1) \cdot f^2(a_2) \cdot \ldots \cdot f^{n-2}(a_{n-2}) \cdot u \tag{15}$$

i.e. u has a left inverse ${}^{-1}u = f(a_1) \cdot f^2(a_2) \cdot \ldots \cdot f^{n-2}(a_{n-2})$. Applying f in (15) we obtain

$$\begin{aligned} a &= f^2(a_1) \cdot f^3(a_2) \cdot \ldots \cdot f^{n-2}(a_{n-3}) \cdot f^{n-1}(a_{n-2}) \cdot f(u) \\ &= f^2(a_1) \cdot \ldots \cdot f^{n-2}(a_{n-3}) \cdot u \cdot a_{n-2}.\end{aligned}$$

Applying f we get

$$a = f^3(a_1) \cdot \ldots \cdot f^{n-2}(a_{n-4}) \cdot u \cdot a_{n-3} \cdot f(a_{n-2})$$

and finally $a = u \cdot a_1 \cdot f(a_2) \cdot \ldots \cdot f^{n-3}(a_{n-2})$ i.e. u has a right inverse $u^{-1} = a_1 \cdot f(a_2) \cdot \ldots \cdot f^{n-3}(a_{n-2})$. Therefore u is invertible.

From (8) we obtain $f^{n-1}(x) = u \cdot x \cdot u^{-1}$.

In conclusion $(A,\alpha) = H_{<f,u>}(A,\cdot)$ and from the proof of Theorem 7 we have $u^{-1} = \bar{a}$.

Let now a be a skewable element in (A,α). Then $(A,\alpha) = H_{<f,u>}(A,\cdot)$, where $(A,\cdot) = \mathrm{ret}_{(a)^{n-2}}(A,\alpha)$, $u = \alpha((\bar{a})^n)$ and $f(x) = \alpha(\bar{a}, x, (a)^{n-2})$.

Theorem 12. *An element x is skewable in (A,α) if and only if it is invertible in $(A,\cdot)$.*

Proof. Let x be skewable. We prove that

$$x^{-1} = \alpha(\bar{a}, \bar{x}, (x)^{n-3}, \bar{a}). \tag{16}$$

We have

$$\begin{aligned} x\cdot x^{-1} &= \alpha(x,(a)^{n-2},\alpha(\bar{a},\bar{x},(x)^{n-3},\bar{a}))\\ &= \alpha(\alpha(x,(a)^{n-2},\bar{a}),\bar{x},(x)^{n-3},\bar{a})\\ &= \alpha(x,\bar{x},(x)^{n-3},\bar{a}) = \bar{a}\end{aligned}$$

and

$$\begin{aligned} x^{-1}\cdot x &= \alpha(\alpha(\bar{a},\bar{x},(x)^{n-3},\bar{a}),(a)^{n-2},x)\\ &= \alpha(\bar{a},\bar{x},(x)^{n-3},\alpha(\bar{a},(a)^{n-2},x))\\ &= \alpha(\bar{a},\bar{x},(x)^{n-3},x) = \bar{a}.\end{aligned}$$

Hence $x\cdot x^{-1} = x^{-1}\cdot x = \bar{a}$ (the unit of $(A,\cdot)$).

Now suppose that x is invertible in $(A,\cdot)$. We prove that x is skewable in (A,α) and $\bar{x}$ verify (16). The equality (16) is equivalent to $x^{-1} = f(\bar{x})\cdot f^2(x)\cdot \ldots \cdot f^{n-2}(x)\cdot u$. Since u is invertible we get $x^{-1}\cdot u^{-1} = f(\bar{x})\cdot f^2(x)\cdot\ldots\cdot f^{n-2}(x)$ which implies $f^{-1}(x^{-1}\cdot u^{-1}) = \bar{x}\cdot f(x)\cdot\ldots\cdot f^{n-3}(x)$ and finally

$$\bar{x} = f^{-1}(x^{-1})\cdot u^{-1}\cdot f^{n-3}(x^{-1})\cdot\ldots\cdot f(x^{-1}) \tag{17}$$

Now we prove that indeed $\bar{x}$ is the skew element to x. From $f^{n-1}(x) = u\cdot x\cdot u^{-1}$ we get $f^{n-1}(x^{-1}) = u\cdot x^{-1}\cdot u^{-1}$ and applying f^{-1} we obtain

$$f^{n-2}(x^{-1}) = u\cdot f^{-1}(x^{-1})\cdot u^{-1} \tag{18}$$

Now

$$\begin{aligned}\alpha(y,(x)^{n-2},\bar{x}) &= y\cdot f(x)\cdot\ldots\cdot f^{n-2}(x)\cdot u\cdot\bar{x}\\ &= y\cdot f(x)\cdot\ldots\cdot f^{n-2}(x)\cdot u\cdot f^{-1}(x^{-1})\cdot u^{-1}\cdot f^{n-3}(x^{-1})\cdot\ldots\cdot f(x^{-1})\\ &= y\cdot f(x)\cdot\ldots\cdot f^{n-2}(x)\cdot f^{n-2}(x^{-1})\cdot f^{n-3}(x^{-1})\cdot\ldots\cdot f(x^{-1}) = y\end{aligned}$$

From (18) we obtain $f^{-1}(x^{-1})\cdot u^{-1} = u^{-1}\cdot f^{n-2}(x^{-1})$ and then $\alpha(\bar{x},(x)^{n-2},y) = f^{-1}(x^{-1})\cdot u^{-1}\cdot f^{n-3}(x^{-1})\cdot \ldots \cdot f(x^{-1})\cdot f(x)\cdot f^2(x)\cdot \ldots \cdot f^{n-3}(x)\cdot f^{n-2}(x)\cdot u\cdot y = f^{-1}(x^{-1})\cdot u^{-1}\cdot f^{n-2}(x)\cdot u\cdot y = u^{-1}\cdot f^{n-2}(x^{-1})\cdot f^{n-2}(x)\cdot u\cdot y = y$.

As a simple consequence we obtain the Gluskin-Hosszú theorem.

Corollary 2. *An n-groupoid is an n-group iff it is H-derived from a group.*

Proof. Follows directly from Theorem 6.

Let now $S(A) = \{x \in A \mid x \text{ is skewable}\}$.

Theorem 13. *If $S(A) \neq \emptyset$ then (A,α) is $S(A)$-cancellative.*

Proof. Let be $s_1,\ldots,s_n \in S(A)$. The equality $\alpha(s_1^{i-1},x,s_{i+1}^n) = \alpha(s_1^{i-1},y,s_{i+1}^n)$ is equivalent to

$$\begin{aligned} & s_1\cdot f(s_2)\cdot \ldots \cdot f^{i-2}(s_{i-1})\cdot f^{i-1}(x)f^i(s_{i+1})\cdot \ldots \cdot f^{n-1}(s_n)\cdot u \\ = \; & s_1\cdot f(s_2)\cdot \ldots \cdot f^{i-2}(s_{i-1})\cdot f^{i-1}(y)\cdot f^i(s_{i+1})\cdot \ldots \cdot f^{n-1}(s_n)\cdot u \end{aligned}$$

Since $s_1,\ldots,s_n$ are invertible in $(A,\cdot)$ and f is an automorphism we get $x = y$.

Theorem 14. *If $x \in S(A)$ then x is infinitely skewable.*

Proof. From the proof of Theorem 8 we get $(A,\alpha) = H_{<f,u>}(A,\cdot)$, where $(A,\cdot) = \mathrm{ret}_{(x)^{n-2}}(A,\alpha)$ and $u = \alpha((\bar{x})^n)$. Now from the proof of Theorem 7 we have $u^{-1} = \overline{(\overline{x})} = \bar{\bar{x}}$, i.e. $\bar{x}$ is skewable. Using this fact we have $(A,\alpha) = H_{<g,v>}(A,+)$, where $(A,+) = \mathrm{ret}_{(\bar{x})^{n-2}}(A,\alpha)$ and $v = \alpha((\bar{x})^{n-2})$. Again by Theorem 7 we get $v^{-1} = \overline{(\bar{\bar{x}})} = \bar{\bar{\bar{x}}}$, etc.

Following Post [10] (see also [3]), we define the *n-ary power* putting

$$x^{<k>} = \begin{cases} \alpha(x^{<k-1>},(x)^{n-1}) & \text{for } k>0 \\ x & \text{for } k=0 \\ y : \alpha(y,x^{<-k-1>},(x)^{n-2}) = x & \text{for } k<0 \end{cases}$$

It is easy to verify that the following exponential laws hold

$$\begin{gathered} \alpha(x^{<s_1>},\ldots,x^{<s_n>}) = x^{<s_1+\ldots+s_{n+1}>}, \\ (x^{<r>})^{<s>} = x^{<rs(n-1)+s+r>} = (x^{<s>})^{<r>} \end{gathered}$$

Let $\bar{x}^{(0)} = x$ and let $\bar{x}^{(k+1)}$ be the skew element to $\bar{x}^{(k)}$, $k \geq 0$. Using the above laws we can see that $\bar{x} = x^{<-1>}$ and, in the consequence $\bar{x}^{(2)} = (x^{<-1>})^{<-1>} = x^{<n-3>}$. Generally, $\bar{x}^{<k>} = x^{<S_k>}$ for $S_k = -\sum_{i=0}^{k-1}(2-n)^i$.

Theorem 15.*If $S(A) \neq \emptyset$ then $(S(A), \alpha)$ is an n-subgroup of (A, α).*

Proof. If $x_1, \ldots, x_n \in S(A)$ they are invertible in $(A, \cdot)$ and $\alpha(x_1^n) = x_1 \cdot f(x_2) \cdot \ldots \cdot f^{n-1}(x_n) \cdot u$ having all factors invertible is invertible too. Hence, by Theorem 12, $\alpha(x_1^n)$ is skewable. Therefore $(S(A), \alpha)$ is an n-subsemigroup of (A, α). Since all elements of $(S(A), \alpha)$ are skewable is an n-group.

We finish by a simple consequence of this theorem.

Corollary 3.*Let $x, x_1, \ldots, x_n \in S(A)$. Then $\overline{\alpha((x)^n)} = \alpha((\bar{x})^n)$ and if (A, α) is abelian $\overline{\alpha(x_1^n)} = \alpha(\bar{x}_1^n)$.*

Proof. These equalities hold in n-groups (see [3], [4]).

References

[1] Dudek, W. A., Glazek, K. and Gleichgewicht, B., A note on the axioms of n-groups, Colloquia Math., Soc. J. Bolyai 29 "Universal Algebra", Esztergom (Hungary) 1977, 192-202. (North-Holland, Amsterdam 1982).

[2] Dudek, W. A. and Michalski, J., On a generalization of Hosszú theorem, Demonstratio Math. 15 (1982), 783-805.

[3] Dudek, W. A., On some old and new problems in n-ary groups, Quasigroups and Related Systems 8 (2001), 15-36.

[4] Glazek, K. and Gleichgewicht, B., Abelian n-groups, Colloquia Math. Soc. J. Bolyai 29 "Universal Algebra", Esztergom (Hungary) 1977, 321-329. (North-Holland, Amsterdam 1982).

[5] Hosszú, M., On the explicit form of n-group operations, Publ. Math. Debrecen 10 (1963), 88-92.

[6] Kuros, A.G., General algebra, Lecture notes, 1969-1970, Nauka, Moscow, 1974 (in Russian).

[7] Monk, J. D. and Sioson, F. M., On the general theory of n-groups, Fund. Math. 72 (1971), 233-244.

[8] Petrescu, A., Polyadic semigroups with skew elements, Univ. "Petru Maior" Târgu-Mureş, Buletin ştiinţific vol. XVII, 2004, 303-308.

[9] Petrescu, A., H-derived polyadic semigroups, Algebra Symposium, Cluj-Napoca, May 27-28, 2005 (to appear in Proceedings of the Algebra Symposium).

[10] Post, E. L., Polyadic groups, Trans. Amer. Math. Soc. 48 (1940), 208-330.

[11] Sokolov, E.I., On the Gluskin-Hosszú theorem for Dörnte n-groups, (Russian), Math. Issled. 39 (1976), 187-189.

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