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GENERALIZED CLOSED SETS IN ČECH CLOSED SPACES

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Abstract. The purpose of the present paper is to introduce the concept of generalized closed sets in Čech closure spaces and investigate some of their characterizations.

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1. Introduction

Generalized closed sets, briefly g-closed sets, in a topological space were introduced by N. Levine [7] in order to extend some important properties of closed sets to a larger family of sets. For instance, it was shown that compactness, normality and completeness in a uniform space are inherited by g-closed subsets. K. Balachandran, P. Sundaram and H. Maki [1] introduced the notion of generalized continuous maps, briefly g-continuous maps, by using g-closed sets and studied some of their properties.

Čech closure spaces were introduced by E. Čech in [2] and then studied by many authors, see e.g. [3], [4], [9] and [10]. In this paper, we introduce generalized closed (g-closed) sets in a Čech closure space. We study unions, intersections and subspaces of g-closed subsets of a Čech closure space. Generalized open (g-open) subsets of Čech closure spaces are also introduced and their properties are studied.

2. Preliminaries

An operator $u : P(X) \to P(X)$ defined on the power set $P(X)$ of a set X satisfying the axioms :

(C1) $u\emptyset = \emptyset$,

(C2) $A \subseteq uA$ for every $A \subseteq X$,

(C3) $u(A \cup B) = uA \cup uB$ for all $A, B \subseteq X$.

is called a *Čech closure operator* and the pair (X, u) is called a *Čech closure space*. For short, the space will be noted by X as well, and called a *closure space*. A closure operator u on a set X is called *idempotent* if $uA = uuA$ for all $A \subseteq X$.

A subset A is *closed* in the Čech closure space (X, u) if $uA = A$ and it is *open* if its complement is closed. The empty set and the whole space are both open and closed.

A Čech closure space (Y, v) is said to be a *subspace* of (X, u) if $Y \subseteq X$ and $vA = uA \cap Y$ for each subset $A \subseteq Y$. If Y is closed in (X, u), then the subspace (Y, v) of (X, u) is said to be closed too.

Let (Y, v) be a Čech closed subspace of (X, u). If F is a closed subset of (Y, v), then F is a closed subset of (X, u).

Let (X, u) and (Y, v) be Čech closure spaces. A map $f : (X, u) \to (Y, v)$ is said to be *continuous* if $f(uA) \subseteq vf(A)$ for every subset $A \subseteq X$.

One can see that a map $f : (X, u) \to (Y, v)$ is continuous if and only if $uf^{-1}(B) \subseteq f^{-1}(vB)$ for every subset $B \subseteq Y$. Clearly, if $f : (X, u) \to (Y, v)$ is continuous, then $f^{-1}(F)$ is a closed subset of (X, u) for every closed subset F of (Y, v).

Let (X, u) and (Y, v) be Čech closure spaces. A map $f : (X, u) \to (Y, v)$ is said to be *closed* (resp. *open*) if $f(F)$ is a closed (resp. open) subset of (Y, v) whenever F is a closed (resp. open) subset of (X, u).

The *product* of a family $\{(X_\alpha, u_\alpha) : \alpha \in I\}$ of Čech closure spaces, denoted by $\prod_{\alpha \in I} (X_\alpha, u_\alpha)$, is the Čech closure space $\big(\prod_{\alpha \in I} X_\alpha, u\big)$ where $\prod_{\alpha \in I} X_\alpha$ denotes the cartesian product of sets X_α, $\alpha \in I$, and u is the Čech closure operator generated by the projections $\pi_\alpha : \prod_{\alpha \in I} (X_\alpha, u_\alpha) \to (X_\alpha, u_\alpha)$, $\alpha \in I$, i.e., is defined by $uA = \prod_{\alpha \in I} u_\alpha \pi_\alpha(A)$ for each $A \subseteq \prod_{\alpha \in I} X_\alpha$.

Clearly, if $\{(X_\alpha, u_\alpha) : \alpha \in I\}$ is a family of Čech closure spaces, then the projection map $\pi_\beta : \prod_{\alpha \in I} (X_\alpha, u_\alpha) \to (X_\beta, u_\beta)$ is closed and continuous for every $\beta \in I$.

Proposition 2.1. *Let $\{(X_\alpha, u_\alpha) : \alpha \in I\}$ be a family of Čech closure spaces and let $\beta \in I$. Then F is a closed subset of (X_β, u_β) if and only if $F \times \prod_{\substack{\alpha \neq \beta \\ \alpha \in I}} X_\alpha$ is a closed subset of $\prod_{\alpha \in I} (X_\alpha, u_\alpha)$.*

Proof. Let F be a closed subset of (X_β, u_β). Since π_β is continuous, $\pi_\beta^{-1}(F)$ is a closed subset of $\prod_{\alpha \in I} (X_\alpha, u_\alpha)$. But $\pi_\beta^{-1}(F) = F \times \prod_{\substack{\alpha \neq \beta \\ \alpha \in I}} X_\alpha$, hence $F \times \prod_{\substack{\alpha \neq \beta \\ \alpha \in I}} X_\alpha$ is a closed subset of $\prod\limits_{\alpha\in I}(X_\alpha, u_\alpha)$.

Conversely, let $F \times \prod\limits_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$ be a closed subset of $\prod\limits_{\alpha\in I}(X_\alpha, u_\alpha)$. Since π_β is closed, $\pi_\beta\Big(F \times \prod\limits_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha\Big) = F$ is a closed subset of (X_β, u_β).

The following statement is evident :

Proposition 2.2. *Let $\{(X_\alpha, u_\alpha) : \alpha \in I\}$ be a family of Čech closure spaces and let $\beta \in I$. Then G is an open subset of (X_β, u_β) if and only if $G \times \prod\limits_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$ is an open subset of $\prod\limits_{\alpha\in I}(X_\alpha, u_\alpha)$.*

3. Generalized closed sets

In this section, we introduce a new class of closed sets in Čech closure spaces and study some of their properties.

Definition 3.1. Let (X, u) be a Čech closure space. A subset $A \subseteq X$ is called a *generalized closed* set, briefly a *g-closed* set, if $uA \subseteq G$ whenever G is an open subset of (X, u) with $A \subseteq G$. A subset $A \subseteq X$ is called a *generalized open* set, briefly a *g-open* set, if its complement is g-closed.

Remark 3.2. Every closed set is g-closed. The converse is not true as can be seen from the following example.

Example 3.3. Let $X = \{1, 2\}$ and define a Čech closure operator u on X by $u\emptyset = \emptyset$ and $u\{1\} = u\{2\} = uX = X$. Then $\{1\}$ is g-closed but it is not closed.

Proposition 3.4. *Let (X, u) be a Čech closure space. If A and B are g-closed subsets of (X, u), then $A \cup B$ is g-closed.*

Proof. Let G be an open subset of (X, u) such that $A \cup B \subseteq G$. Then $A \subseteq G$ and $B \subseteq G$. Since A and B are g-closed, $uA \subseteq G$ and $uB \subseteq G$. Consequently, $u(A \cup B) = uA \cup uB \subseteq G$. Therefore, $A \cup B$ is g-closed.

The intersection of two g-closed sets need not be a g-closed set as can be seen from the following example.

Example 3.5. Let $X = \{a, b, c\}$ and define a Čech closure operator u on X by $u\emptyset = \emptyset$ and $u\{a\} = \{a, b\}$, $u\{b\} = u\{c\} = u\{b, c\} = \{b, c\}$ and $u\{a, b\} = u\{a, c\} = uX = X$. Then $\{a, b\}$ and $\{a, c\}$ are g-closed but $\{a, b\}\cap\{a, c\} = \{a\}$ is not g-closed.

Proposition 3.6. *Let (X, u) be a Čech closure space. If A is g-closed and F is closed in (X, u), then $A \cap F$ is g-closed.*

Proof. Let G be an open subset of (X, u) such that $A \cap F \subseteq G$. Then $A \subseteq G \cup (X - F)$ and so $uA \subseteq G \cup (X - F)$. Then $uA \cap F \subseteq G$. Since F is closed, $u(A \cap F) \subseteq G$. Hence, $A \cap F$ is g-closed.

Proposition 3.7. *Let (Y, v) be a closed subspace of (X, u). If F is a g-closed subset of (Y, v), then F is a g-closed subset of (X, u).*

Proof. Let G be an open subset of (X, u) such that $F \subseteq G$. Then $F \subseteq G \cap Y$. Since F is g-closed and $G \cap Y$ is open in (Y, v), $uF \cap Y = vF \subseteq G$. But Y is a closed subset of (X, u) and $uF \subseteq G$. Hence, F is a g-closed subset of (X, u).

The following statement is obviuos :

Proposition 3.8. *Let (X, u) be a Čech closure space and let $A \subseteq X$. If A is both open and g-closed, then A is closed.*

Proposition 3.9. *Let (X, u) be a Čech closure space and let u be idempotent. If A is a g-closed subset of (X, u) such that $A \subseteq B \subseteq uA$, then B is a g-closed subset of (X, u).*

Proof. Let G be an open subset of (X, u) such that $B \subseteq G$. Then $A \subseteq G$. Since A is g-closed, $uA \subseteq G$. As u is idempotent, $uB \subseteq uuA = uA \subseteq G$. Hence, B is g-closed.

Proposition 3.10. *Let (X, u) be a Čech closure space and let $A \subseteq X$. If A is g-closed, then $uA - A$ has no nonempty closed subset.*

Proof. Suppose that A is g-closed. Let F be a closed subset of $uA - A$. Then $F \subseteq uA \cap (X - A)$ and so $A \subseteq X - F$. Consequently, $F \subseteq X - uA$. Since $F \subseteq uA$, $F \subseteq uA \cap (X - uA) = \emptyset$, thus $F = \emptyset$. Therefore, $uA - A$ contains no nonempty closed set.

The converse of the previous proposition is not true as can be seen from the following example.

Example 3.11. Let $X = \{1, 2, 3\}$ and define a Čech closure operator u on X by $u\emptyset = \emptyset$ and $u\{1\} = \{1, 2\}$, $u\{2\} = u\{3\} = u\{2, 3\} = \{2, 3\}$ and $u\{1, 2\} = u\{1, 3\} = uX = X$. Then $u\{1\} - \{1\} = \{2\}$ does not contain nonempty closed set. But $\{1\}$ is not g-closed.

Corollary 3.12. *Let (X, u) be a Čech closure space and let A be a g-closed subset of (X, u). Then A is closed if and only if $uA - A$ is closed.*

Proof. Let A be a g-closed subset of (X, u). If A is closed, then $uA - A = \emptyset$. But $\emptyset$ is always closed. Therefore, $uA - A$ is closed.

Conversely, suppose that $uA - A$ is closed. As A is g-closed, $uA - A = \emptyset$ by Proposition 3.10. Consequently, $uA = A$. Hence, A is closed.

Proposition 3.13. *Let (X, u) be a Čech closure space and let u be idempotent. If A is g-closed and $A \subseteq B \subseteq uA$, then $uB - B$ has no nonempty closed subset.*

Proof. $A \subseteq B$ implies $X - B \subseteq X - A$ and $B \subseteq uA$ implies $uB \subseteq uuA = uA$. Thus $uB \cap (X - B) \subseteq uA \cap (X - A)$ which yields $uB - B \subseteq uA - A$. As A is g-closed, $uA - A$ has no nonempty closed subset. The same must be true for $uB - B$.

Proposition 3.14. *Let (X, u) be a Čech closure space. A set $A \subseteq X$ is g-open if and only if $F \subseteq X - u(X - A)$ whenever F is closed and $F \subseteq A$.*

Proof. Suppose that A is g-open and let F be a closed subset of (X, u) such that $F \subseteq A$. Then $X - A \subseteq X - F$. But $X - A$ is g-closed and $X - F$ is open. It follows that $u(X - A) \subseteq X - F$ and hence $F \subseteq X - u(X - A)$.

Conversely, let G be an open subset of (X, u) such that $X - A \subseteq G$. Then $X - G \subseteq A$. Since $X - G$ is closed, $X - G \subseteq X - u(X - A)$. Consequently, $u(X - A) \subseteq G$. Hence, $X - A$ is g-closed and so A is g-open.

The union of two g-open sets need not be a g-open set as we can see in Example 3.5 : Put $A = \{b\}$ and $B = \{c\}$ Then A and B are g-open but $A \cup B = \{b, c\}$ is not g-open.

Proposition 3.15. *Let (X, u) be a Čech closure space. If A is g-open and B is open in (X, u), then $A \cup B$ is g-open.*

Proof. Let F be a closed subset of (X, u) such that $F \subseteq A \cup B$. Then $X - (A \cup B) \subseteq X - F$. Hence, $(X - A) \cap (X - B) \subseteq X - F$. By Proposition 3.6, $(X-A)\cap(X-B)$ is g-closed. Therefore, $u((X-A)\cap(X-B)) \subseteq X-F$. Consequently, $F \subseteq X - u((X - A) \cap (X - B)) = X - u(X - (A \cup B))$. By Proposition 3.14, $A \cup B$ is g-open.

Proposition 3.16. *Let (X, u) be a Čech closure space. If A and B are g-open subsets of (X, u), then $A \cap B$ is g-open.*

Proof. Let F be a closed subset of (X, u) such that $F \subseteq A \cap B$. Then $X - (A \cap B) \subseteq X - F$. Consequently, $(X - A) \cup (X - B) \subseteq X - F$. By Proposition 3.4, $(X - A) \cup (X - B)$ is g-closed. Thus, $u((X - A) \cup (X - B)) \subseteq X - F$, hence $F \subseteq X - u((X - A) \cup (X - B)) = X - (X - (A \cap B))$. By Proposition 3.14, $A \cap B$ is g-open.

Proposition 3.17. *Let (X, u) be a Čech closure space. If A is a g-open subset of (X, u), then $G = X$ whenever G is open and $(X - u(X - A)) \cup (X - A) \subseteq G$.*

Proof. Suppose that A is g-open. Let G be an open subset of (X, u) such that $(X - u(X - A)) \cup (X - A) \subseteq G$. Then $X - G \subseteq X - ((X - u(X - A)) \cup (X - A))$. Therefore, $X - G \subseteq u(X - A) \cap A$ or, equivalently, $X - G \subseteq u(X - A) - (X - A)$.

But $X-G$ is closed and $X-A$ is g-closed. Thus, by Proposition 3.10, $X-G=\emptyset$. Consequently, $X=G$.

The converse of this proposition is not true as can be seen from Example 3.11 : Put $A=\{2,3\}$. Then A is not g-open and $(X-u(X-A))\cup(X-A)=\{3\}\cup\{1\}\subseteq G$ gives $G=X$. But A is not g-open.

Proposition 3.18. *Let (X,u) be a Čech closure space and let $A\subseteq X$. If A is a g-closed, then $uA-A$ is g-open.*

Proof. Suppose that A is g-open. Let F be a closed subset of (X,u) such that $F\subseteq uA-A$. By Proposition 3.10, $F=\emptyset$ and hence $F\subseteq X-u(X-(uX-A))$. By Proposition 3.14, $uA-A$ is g-open.

The converse of this result is not true as can be seen from Example 3.11 : Put $A=\{1\}$. Then $u\{1\}-\{1\}=\{2\}$ which is g-open. But $\{1\}$ is not g-closed.

Proposition 3.19. *Let $\{(X_\alpha,u_\alpha):\alpha\in I\}$ be a family of Čech closure spaces and let $\beta\in I$. Then G is a g-open subset of (X_β,u_β) if and only if $G\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in I}}X_\alpha$ is a g-open subset of $\prod_{\alpha\in I}(X_\alpha,u_\alpha)$.*

Proof. Let F be a closed subset of $\prod_{\alpha\in I}(X_\alpha,u_\alpha)$ such that $F\subseteq G\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in I}}X_\alpha$. Then $\pi_\beta(F)\subseteq G$. Since $\pi_\beta(F)$ is closed and G is g-open in (X_β,u_β), $\pi_\beta(F)\subseteq X_\beta-u_\beta(X_\beta-G)$. Therefore, $F\subseteq\pi_\beta^{-1}(X_\beta-u_\beta(X_\beta-G))=\prod_{\alpha\in I}X_\alpha-\prod_{\alpha\in I}u_\alpha\pi_\alpha\Big(\prod_{\alpha\in I}X_\alpha-G\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in I}}X_\alpha\Big)$. By Proposition 3.14, $G\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in I}}X_\alpha$ is a g-open subset of $\prod_{\alpha\in I}(X_\alpha,u_\alpha)$.

Conversely, let F be a closed subset of (X_β,u_β) such that $F\subseteq G$. Then $F\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in I}}X_\alpha\subseteq G\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in I}}X_\alpha$. Since $F\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in I}}X_\alpha$ is closed and $G\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in I}}X_\alpha$ is g-open in $\prod_{\alpha\in I}(X_\alpha,u_\alpha)$, $F\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in I}}X_\alpha\subseteq\prod_{\alpha\in I}X_\alpha-\prod_{\alpha\in I}u_\alpha\pi_\alpha\Big(\prod_{\alpha\in I}X_\alpha-G\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in I}}X_\alpha\Big)$ by Proposition 3.14. Therefore, $\prod_{\alpha\in I}u_\alpha\pi_\alpha\Big((X_\beta-G)\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in I}}X_\alpha\Big)\subseteq\prod_{\alpha\in I}X_\alpha-F\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in I}}X_\alpha=(X_\beta-F)\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in I}}X_\alpha$. Consequently, $u_\beta(X_\beta-G)\subseteq X_\beta-F$ implies $F\subseteq X_\beta-u_\beta(X_\beta-G)$. Hence, G is a g-open subset of (X_β,u_β).

Proposition 3.20. *Let $\{(X_\alpha,u_\alpha):\alpha\in I\}$ be a family of Čech closure spaces and let $\beta\in I$. Then F is a g-closed subset of (X_β,u_β) if and only if $F\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in I}}X_\alpha$ is* *a g-closed subset of* $\prod_{\alpha\in I}(X_\alpha, u_\alpha)$.

Proof. Let F be a g-closed subset of (X_β, u_β). Then $X_\beta - F$ is a g-open subset of (X_β, u_β). By Proposition 3.19, $(X_\beta - F) \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha = \prod_{\alpha\in I} X_\alpha - F \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$ is a g-open subset of $\prod_{\alpha\in I}(X_\alpha, u_\alpha)$. Hence, $F \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$ is a g-closed subset of $\prod_{\alpha\in I}(X_\alpha, u_\alpha)$.

Conversely, let G be an open subset of (X_β, u_β) such that $F \subseteq G$. Then $F \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha \subseteq G \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$. Since $F \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$ is g-closed and $G \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$ is open in $\prod_{\alpha\in I}(X_\alpha, u_\alpha)$, $\prod_{\alpha\in I} u_\alpha\pi_\alpha\Big(F \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\beta\Big) \subseteq G \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$. Consequently, $u_\beta F \subseteq G$. Therefore, F is a g-closed subset of (X_β, u_β).

Proposition 3.21. *Let* $\{(X_\alpha, u_\alpha) : \alpha \in I\}$ *be a family of Čech closure spaces. For each* $\beta \in I$*, let* $\pi_\beta : \prod_{\alpha\in I}(X_\alpha, u_\alpha) \to (X_\beta, u_\beta)$ *be the projection map. Then*

(i) If F *is a g-closed subset of* $\prod_{\alpha\in I}(X_\alpha, u_\alpha)$*, then* $\pi_\beta(F)$ *is a g-closed subset of* (X_β, u_β).

(ii) If F *is a g-closed subset of* (X_β, u_β)*, then* $\pi_\beta^{-1}(F)$ *is a g-closed subset of* $\prod_{\alpha\in I}(X_\alpha, u_\alpha)$.

Proof. (i) Let F be a g-closed subset of $\prod_{\alpha\in I}(X_\alpha, u_\alpha)$ and let G be an open subset of (X_β, u_β) such that $\pi_\beta(F) \subseteq G$. Then $F \subseteq \pi_\beta^{-1}(G) = G \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$. Since F is g-closed and $G \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$ is open, $\prod_{\alpha\in I} u_\alpha\pi_\alpha(F) \subseteq G \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$. Consequently, $u_\beta\pi_\beta(F) \subseteq G$. Hence, $\pi_\beta(F)$ is a g-closed subset of (X_β, u_β).

(ii) Let F be a g-closed subset of (X_β, u_β). Then $\pi_\beta^{-1}(F) = F \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$. By Proposition 3.20, $F \times \prod_{\substack{\alpha\neq\beta\\ \alpha\in I}} X_\alpha$ is a g-closed subset of $\prod_{\alpha\in I}(X_\alpha, u_\alpha)$. Therefore, $\pi_\beta^{-1}(F)$ is a g-closed subset of $\prod_{\alpha\in I}(X_\alpha, u_\alpha)$.

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