DISCOVERING A RULE AND ITS MATHEMATICAL JUSTIFICATION IN MODELING ACTIVITIES USING SPREADSHEET

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The present study has its objective in describing how students discover rules underlying a certain phenomenon at question and the process in which they mathematically justify such discovery by way of utilizing tables and graphs in use of the cell reference function of spreadsheet when a problem is not easily solved through simple symbol manipulation in a paper and pencil environment.

INTRODUCTION AND THEORETICAL FRAMEWORK

Mathematical modeling activities carry significance in that they offer students the experience of discovering and constructing their own mathematical knowledge, and in that they help students realize the applicability and necessity of mathematics. Generally modeling activities start from real-life situations, and therefore naturally accompany a great deal of complicated calculation and the discovery of patterns or rules implied within certain situations in order to understand and predict the problem situation. In such cases spreadsheet could be usefully used.

Not only is spreadsheet easy to use, but it can also provoke student-centered, discovery-centered learning (Beare, 1993; Beare & Hewitson, 1996; Baker & Sugden, 2003). In the course of utilizing spreadsheet, the interaction between fellow students, teacher and student, and computer and student becomes active (Hershkowitz, Dreyfus, Ben-Zvi, Friedlander, Hadas, Resnick & Tabach, 2002), and as students are enabled to design and manipulate more dynamic and visual spreadsheet models they are also able to explore diverse aspects of a phenomenon (Molyneux- Hodgson, Rojano, Sutherland & Ursini, 1999; Neuwirth & Arganbright, 2004).

Such distinctive features of spreadsheet serve as significant factors when students are carrying out modeling activities in small groups. When spreadsheet is used such mathematical models as tables and graphs that are necessary in carrying out modeling activities can be easily constructed, and they are also conducive in enhancing the user’s intuition in developing and using algorithms and models necessary for solving mathematical problems (Masalski, 1990). In particular, the cell reference function of spreadsheet that when the value of referred cell is changed then the value in a cell in reference is changed and, consequently the related table and graph also are changed, makes students’ exploration of a problem situation slightly more complicated and dynamic.
The present study attempts to look into how students discover rules inherent in problem situations and how they mathematically justify such discovery in modeling activities using spreadsheet.

**METHODOLOGY**

Class experiments were conducted with 6 tenth grade students that were divided into two groups comprised of three students each. Only one of these students had prior experience in using spreadsheet. In terms of level of academic achievement, 2 were in the upper group of their class, 2 in the middle-upper, and 2 in the middle group of their classes. Each group was heterogeneously comprised according to which class the student came from and the students’ level of academic achievement. The teacher was a male teacher with 9 years’ teaching experience who had utilized spreadsheet for administrative purposes as recording grades. He had never used spreadsheet in his mathematics classes.

Each group that was comprised of three students was provided with one computer. The worksheets that were given in the experiment were divided into two parts: problem solving and reflection. The students were required to write down the problem solving process from partial to whole, and also particular difficulties they went through or feelings they had in using spreadsheet. The experimental class first attempted to solve the problems in a paper and pencil environment and then when the problems were not able to be solved students were asked to use spreadsheet so that they could solve the problem situation.

In exception of the 2 hours allocated for getting used to preliminary activities, the experimental classes were conducted 8 times, 2 hours each over approximately two months. The spreadsheet that was used for the experiment was Microsoft Excel 2003. Data analysis was carried out by comparing the recorded video capture files and the recorded audio files that were taken in each group. Analysis was conducted using a triangulation based on transcript, the video images, worksheets and survey data. The results that are presented here came from the 7th class using the problem situation suggested by Heid (1997, p. 96) (See Figure 1).

**STUDENT IMPLEMENTATION AND ANALYSIS**

The two groups of students went through a similar process of problem solving and also obtained similar results. The activities of one group will be presented here.

**Discovery**

Before using spreadsheet for problem 1, the students were able to establish such equations as \((1040 \times 0.95) + 52 = x_1\), \((x_1 \times 0.95) + 52 = x_2\), where \(x_n\) is the amount of chlorine after \(n\) days, but were not able to move forward. When the teacher recommended to use spreadsheet the students made tables and graphs, and found out through observation that when the initial amount of chlorine put in is 1040g and 52g of chlorine is put in everyday, then the amount of chlorine continues to stay at 1040g and
There is a swimming pool that is 50 m in length, 21 m in width and 1.8 m in depth. There are bacteria spread out in the swimming pool and the manager is required to use chlorine so that the bacteria do not increase in further number. However if the leftover amount of chlorine is excessive it could cause a certain smell particular to chlorine and also stimulation to human skin. Therefore the amount of chlorine needs to be controlled to maintain 0.4~0.6ppm (mg/L). It should be noted that the chlorine put into the water is disappeared by 5% everyday.

1. If the swimming pool manager puts in 1040g at first and then puts in 52g of chlorine everyday, what concentration could the swimming pool maintain? If the initial amount of chlorine put in was changed to 850g, 1100g, 1300g respectively, then how would the concentration change according to the passing of time?

2. When the initial amount of chlorine is 1040g, and the amount of chlorine put in everyday is 40g, 60g, how would the amount and concentration of the chlorine change? In addition, discuss what kind of changes would occur when the initial amount of chlorine is changed and the reasons to such consequences.

Figure 1. Chlorine in the swimming pool problem

the concentration is approximately 0.550ppm. They also found that in the case where the initial amount of chlorine was adjusted to be less than 1040g, the amount continuously increased to come closer to 1040g, and in the case that the amount exceeded 1040g then the amount continuously decreased to come closer to 1040g.

The students were able to draw similar results in problem 2. Figure 2 displayed below indicates the changes in the amount and concentration of chlorine when the daily amount put in is 60g, and when the initial amount is respectively 1040g and 1300g. In Figure 2, the vertical axes of the graphs on the left indicate the concentration of the chlorine and those of the graphs on the right indicate the amount of chlorine. The horizontal axes of all graphs indicate the date. The upper two graphs in above and below of Figure 2 display the results of the first 30 days, and when the students were not able to accurately identify the flow of the changes in chlorine, they increased the number of days as in the two graphs set in the bottom in above and below of Figure 2, making the number of days 367, and drew graphs. The students were able to look for themselves the continuous increase or decrease of the amount and concentration of chlorine and then its maintenance with the passing of time, utilizing tables and graphs. Figure 2 is what Won-jin made in the course of carrying out the modeling activities and captured by the video capture software.

The following excerpt shows the process in which the students first explored the case when the daily amount of chlorine inserted was 60g, and they start to discover that there is a rule underlying between the amount of chlorine put in everyday and the remaining amount of chlorine in problem 2.
Figure 2: Changes in the concentration and amount of chlorine with initial amount 1040g (above) and 1300g (below)

1.1 Won-jin: The manager is putting in 60g everyday. That’s what’s important.

1.2 Ah-young: That’s what’s important. When it’s 60…

1.3 Ah-young: According to how much chlorine is put in everyday, isn’t it that the amount of chlorine is fixed? I mean, don’t you think there’s a pair?

1.4 Jung-yoon: Yeah, right. The pair of this…

1.5 Ah-young: The pair of this one is 52g.(indicating the initial amount of chlorine 1040g)

1.6 Jung-yoon: We can multiply it by 20. Multiply this by 20.(indicating that 60 multiplied by 20 is 1200)
1.7 Won-jin: That’s right. With this. This.
1.9 Won-jin: We discovered a relation. We discovered something amazing. Wow wow wow.

1.10 Won-jin: It’s right. Ha Ha. The amount multiplied by 20 is maintained.
1.11 Teacher: Why 20?
1.12 Won-jin: Because it’s 0.5% of this. In other words, the 0.5% of the amount, I mean, when the amount is 5% of the amount then it is maintained. (indicating the initial amount)
1.13 Jung-yoon: So 40 multiplied by 20 is 800. (The students had just explored the case when 40 replaced 60)

The students found out that as in (1.1), what determined the remaining amount of chlorine or concentration was the 60g of chlorine that was put in everyday. Furthermore, Ah-young was able to roughly assume the relationship between the fixed amount of chlorine and the amount put in everyday (1.3). Jung-yoon found out that when the daily amount put in was multiplied by 20 then a steady amount of chlorine would be produced (1.6). Won-jin realized that this was related to the 5% of chlorine that disappeared everyday (1.12). (1.8), (1.9) and (1.10) show the students rejoicing to such discovery.

In the course of solving these problems the students were able to understand that the remaining amount of chlorine and its concentration after a certain amount of time depend only on the daily amount of chlorine put in. They were also able to identify the relationships within the pairs of amounts comprising the initial amount and the daily amount, 1040g and 52g, 800g and 40g, and also 1200g and 60g. The reason the students were able to easily observe the relationship lied in the fact that they could dynamically change the graph using the cell reference function. In the same way the students were able to confirm that even in different cases when the daily amount of chlorine was neither 40g nor 60g, the amount of chlorine inched closer to 20 times the amount that was put in daily. It is worthy of note that the process of discovering a rule, the interaction among fellow students and computer was active and the teacher played a role of catalyst.

**Justification**

In order to justify whether or not the chlorine remains at 20 times the amount put in daily, the teacher introduced the formula \( (x_n \times 0.95) + 40 = x_{n+1} \) to students. He wanted students to understand that they might set \( x_n = x_{n+1} \) for sufficiently large \( n \) and those values could be denoted by a single letter. This understanding is important for students who have not yet learned the convergence of sequences and limits to interpret their discovery mathematically. The following excerpt shows that the teacher’s role of intervention over a role of catalyst.
2.1 Teacher: We want to find the value which becomes constant when \( n \) becomes sufficiently large. What do the values of \( x_n \) and \( x_{n+1} \) become when \( n \) is sufficiently large?

2.2 Jung-yoon: Become closer.

2.3 Won-jin: I don’t know well. By the way there is no criterion for the largeness. And the difference between \( x_n \) and \( x_{n+1} \) is big when \( n \) is small.

2.4 Teacher: Well, what do you think about what we know? What do the values become when \( n \) is sufficiently large?

2.5 Won-jin: Become close.

2.6 Teacher: You know that the values become almost the same. We want to get it.

Students accepted that \( x_n \) and \( x_{n+1} \) are same when \( n \) is sufficiently large while the values \( x_n \)'s were different when \( n \) was small. The observation of Figure 3 seems to be a help to the acceptance.

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Figure 3: The amount and concentration of chlorine when the number of days is small (above) and large (below)

There were some difficulties in admitting that they might set all values \( x_n \)'s can be denoted by a single letter, but they came to such conclusion by the teacher’s intervention. This means the students who have not learned the concept of limit formally could accept it intuitively by the informal approach through table and graph and teacher’s proper intervention.

Figure 4 shows Jung-yoon’s justification process when the amount of chlorine is respectively 40g and 60g. When the daily amount is 40g and \( n \) is large enough, Jung-
yoon utilized $x_n = x_{n+1}$ and came up with $x_n = x_{n+1} = (x_n \times 0.95) + 40$ and obtained the value $x_n = 800$.

Similarly Won-jin also put the amount of fixed chlorine as $Y$ when $n$ became large enough, and when 40g, put the equation as $Y \times 0.95 + 40 = Y$ and found that $Y = 800$.

![Figure 4: Jung-yoon’s mathematical justification](image)

At first the students did not know the reason why their teacher introduced the formula $(x_n \times 0.95) + 40 = x_{n+1}$ to justify that the remaining amount of chlorine is 20 times the amount put in daily, but later they understood the reason as the following answer given by Jung-yoon shows.

3.1 Teacher: Why do you multiply such 20?

3.2 Jung-yoon: When I simplify this. (pointing the formula $(5/100) \times x_n = 40$ in Figure 4).

The other students also understood the reason and finally the fact that the multiplier depends on the rate of the disappearance of chlorine.

When asked to express her impression after using Excel in modeling activities, Ah-young said that she was able to find rules of problems requiring convoluted calculation using Excel, and in particular said that she was able to find out that the shape of the graph changed according to how much chlorine was daily put in. Won-jin explained that he could find out information relating to many different values by looking at the graph and added that math class would be more interesting if such software could be used in class. Considering the students’ reaction it is understood that Excel facilitates the exploration of rules inherent in problem situations and that students appreciated that Excel helps mathematics to be more fun and interesting.
CONCLUSION

In learning through modeling activities, finding and justifying the patterns and rules embedded in daily life is necessary. The students presented here were able to determine that the amount and concentration of chlorine according to time did not depend on the initial amount put in but depended on the daily amount put in. They were also able to identify relationships between them and ultimately obtain mathematical justification.

Even in a pencil and paper environment, students were able to obtain basic recurrence formulas, but it was hard for them to understand the meanings implied in the algebraic models. In a spreadsheet environment they were able to freely deal such recurrence formulas and convert them into tables, and were also able to more accurately understand the meanings of them. The tables and graphs produced by the students could work as a mental tool to make them to form the mathematical concepts like limits of sequences intuitively. And students could use this tool to justify the rule they found mathematically.

In the discovering process the interaction among students with spreadsheet became more active than the teacher’s role, but in the course of justifying process the teacher’s role became more active than the interaction among students.

References


