

RESEARCH ON THE PROCESS OF UNDERSTANDING MATHEMATICS: THE INCLUSION RELATION AMONG FRACTIONS, DECIMALS AND WHOLE NUMBERS

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In the research on the process of understanding mathematics, Koyama (1992) presented the so-called “two-axes process model” of understanding mathematics as a useful and effective framework for mathematics teachers. The model consists of two axes, i.e. the vertical axis implying levels of understanding such as mathematical entities, relations of them, and general relations, and the horizontal axis implying three learning stages of intuitive, reflective, and analytic at each level. By analyzing elementary school mathematics classes in Japan, Koyama (2000, 2002, 2003) suggested that a teacher should make a plan of teaching and learning mathematics in the light of “two-axes process model”, and that she/he should play a role as a facilitator for the dialectic process of individual and social constructions.

The purpose of this research is to examine closely the 40 fifth-graders’ process of understanding the inclusion relation among fractions, decimals and whole numbers in a classroom at the national elementary school attached to Hiroshima University. Up to the fourth grade, these students had learned whole numbers, decimals and fractions. In order to improve their understanding of those numbers and promote their mathematical thinking, with a classroom teacher, we planned the teaching unit of “Fractions” and in total of 10 forty-five minutes’ classes were allocated for the unit in the light of “two-axes process model”. Throughout the classes we encouraged students to think mathematically the inclusion relation among fractions, decimals and whole numbers. The data were collected in the way of observation and videotape-record during these classes, and analyzed it qualitatively to see the change of students’ thinking and the dialectic process of individual and social constructions through discussion among them with their teacher in the classroom. First, as a result of introducing the frame \triangle/\bigcirc for their individual activities, by putting some whole numbers into the frame, students could make different fractions and classified them into three different categories. Second, as a result of the qualitative analysis of students’ discussion, we found that students were interested in thinking the inclusion relation among fractions, decimals and whole numbers, and that they could explore the mathematical reason why some fractions were not changed into finite decimals or whole numbers.

References

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